Department of Economics

Essays on Environmental Economics

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Chapter 1

Introduction

The issue of climate change and the question of whether stringent abatement policy is to be prescribed has been one of the most challenging debates of our generation. Most would agree that climate variability and future climate change impacts will increase the vulnerability of societies around the world. Our recent experience has shown an increase in the frequency as well as the severity of weather events with the recent examples of hurricane Katrina and hurricane Sandy. The impacts will be severe especially in developing countries but also in high risk areas in developed countries where the effect is reinforced by the popularity of these areas, such as Florida. The economic costs of impacts of climate change have been estimated to be several percentages of GDP if no measures are taken either to adapt to or mitigate the effects of climate change. However, there is an ongoing debate regarding the most cost efficient and effective climate change policy. Despite the alarming warnings, there are many who argue for a less stringent policy in favor of waiting for more information that will become available in the future and that could potentially decrease the cost and increase the effectiveness of any policy. On the other hand, they are many who emphasize the potential irreversible effects of climate change and call for immediate intervention. In this spirit, cost benefit analysis and Integrated Assessment Models aim to answer to the controversial question of whether stringent abatement policy is the right prescription. However, a review of this strand literature reveals that the conclusions and policy implications of these models are highly dependent on the assumptions made regarding the parameters and the functional forms.

In this spirit, the first paper of this Thesis aims to cast light on this issue by extending the model used by Pindyck [81] by allowing for different specifications for the damage and utility functions. Using the distribution for temperature change and the economic impact provided by Pindyck [81] which is based on information from the IPCC (2007) and recent IAMs, I estimate a simple measure of the "willingness to pay": the fraction of consumption, $w^*(\tau)$, that society would be willing to sacrifice today and throughout the future to ensure that temperature change at some horizon H would be limited to τ. The value of $w^*(\tau)$ will depend on the distributions of temperature change and the economic impact as well as the utility function assumed and the values for the ethical parameters. Pindyck, as well as most of the IAMs, uses a CRRA "multiplicative " functional form utility which implicitly assumes perfect substitutability between material consumption and environmental amenities. However, as Tol et al [38] have pointed out, the individual derives utility from environmental amenities, not necessarily translated to market consumption. As Weitzman [101] has pointed out, the choice of an additive rather than a multiplicative form for the utility function accounts for this fact and leads to significantly different implications about climate change policy when consumption and temperature change is high, with the additive form resulting to larger values for WTP. Therefore, I employ an "additive" as well as a "multiplicative" form for the utility function and the corresponding damage function applied to the level of consumption rather than growth while allowing for an exponent of $N = 2$ and $N = 3$. Given the current " state of knowledge" Pindyck obtains estimates for the WTP which are generally below 3% even for τ around to $2 - 3°C$. As he states, this is because there is limited weight in the tails of the calibrated distributions for ΔT and the growth rate impact. Instead, the specifications of the model I consider, lead to significantly higher estimations for the WTP and in some extreme cases to a value close to 1. The results are even stronger in the case of a larger excepted temperature change where even a quadratic exponential loss function applied to the level of consumption gives a higher WTP than the benchmark specification of Pindyck. Although it is hard to argue which is the "right" functional form, the discussion is aimed to point out that a seemingly arcane theoretical distinction between a "multiplicative" and an "additive" functional form can have very different implications for the optimal climate change policy. Deep structural uncertainties inherent in the climate change science require a careful treatment in order for the analysis to lead to robust results. Moreover, a more realistic model that accounts for parametric as well as intrinsic uncertainty in the form of exogenous shocks and risks needs to be developed, if we are to safely answer the question of whether a stringent abatement policy needs to be employed.

The BP Deepwater Horizon oil spill of 2010 has focused considerable attention on the potential liability and the operating conduct of big oil companies. Much less attention has been paid, however, to the nature and scope of insurance covering losses caused by catastrophic environmental disasters such as oil spills. In the case of BP, its unusually deep pockets, guaranteed through its diversified portfolio, made full compensation to the victims feasible. However, the real question is how to create incentives for the parties involved in activities with potential environmental consequences, to take actions to prevent such events. The second paper presented in this Thesis aims to answer this question. I model a class of general equilibrium economies with uncertainty, where the probability of each state rather than being exogenous, depends on the level of effort exerted by one agent and is therefore endogenously determined in equilibrium. In a setup where "effort" is costly, I show that generically in the space of endowments, every equilibrium is Pareto inefficient as in all economies with imperfections, an "externality" in our case. Then, I focus on welfare improving policies and in particular on the imposition of "participation"constraints in the financial markets. I show that generically in the space of endowments, there is a Pareto-improving policy in the form of a reallocation of existing assets. Moreover, the results extend to economies with aggregate uncertainty and complete markets as well as to economies with uninsurable idiosyncratic risk. This has the important implication that complete markets need not be optimal in an economy with probability externalities: restricting the insurance opportunities of the agents in our model is welfare improving for the society as a whole. Since big oil companies do not have full insurance against their potential pollution liability but are instead "self- insured " by being sufficiently diversified in their portfolio investments, the above analysis suggests that a less diversified portfolio could make the society as a whole better off.

The role of public policy in environmental problems is a hotly debated issue among economists and non economists alike. The only consensus seems to be that if there is to be public intervention it has to come as an incentive-based instrument rather than command and control. The early "Pigouvian" literature on externalities and corrective taxes has argued that it would be sufficient to levy a tax on an externality generating activity equal to its social marginal damage. However, Pigou taxes would be sufficient to achieve Pareto optimality only in a first best environment. In a second best setting, where there are other distortionary taxes in the system, this prescription must be modified. The problem was originally studied by Sandmo [88] and more recently by Bovenberg and Van der Ploeg [12]. What they have shown is what Sandmo called the "additivity property" where the presence of externality only changes the tax formula for the externality generating good but leaves the optimal income and commodity taxes unaffected. Bovenberg and Van der Ploeg [13] and Bovenberg and de Mooiz [12] conclude that "in the presence of pre-existing distortionary taxes, the optimal pollution tax typically lies below the Pigouvian tax...". In other words, while the first best rule calls for a tax that is only corrective, the second best tax embodies both corrective and optimal tax objectives, a tax structure which may be described as "Ramsey plus Pigou". Following the above literature, Cremer, Gahvari and Ladoux [24] and Cremer, Gahvari and Ladoux [25] allow for more than one polluting goods and for non linear taxation casting light on the importance and the relevance of the separability restrictions on preferences required for the above result and concluding that the optimal environmental levy amy exceed, fall short of, or be equal to the Pigouvian tax depending on the assumptions made.

Geanakoplos and Polemarchakis [40] show that, generically in the space of exchange economies with linear, separable consumption externalities, competitive equilibria are constrained inefficient in the sense that there exists a policy intervention, in the form of anonymous taxes on the purchases of commodities, and anonymous, uniform transfers of revenue, which are balanced from a fiscal point of view and induce welfare improvements for all agents in the economy.

The purpose of the third paper presented in this Thesis is to apply the results of Geanakoplos and Polemarchakis [40] to the more stylized framework used in environmental economics, in order to bring to the attention of environmental economists the more recent developments on market failure theory. This will require us to extend the original results to production economies, while adding much generality to the framework used, for instance, by Sandmo. Importantly, we will also study the case of externalities in an OLG with heterogeneous agents, setting, extending and applying the results of Carvajal and Polemarchakis [16]. We present a benchmark model of an infinite-horizon production economy where production activities generate pollution. After solving for the competitive equilibrium, we show that as in all economies with externalities, the equilibrium is Pareto inefficient. We then focus on the design of the optimal fiscal policy for the implementation of the first-best. We find that a mix of tax on capital and labor, combined with a subsidy on afforestation and lump-sum transfers implement the first-best allocation. The next step is to extend this model to allow for heterogeneous agents and an OLG setup. In such a framework, the lack of information makes the decentralization of the first best infeasible, an argument which has been often used by supporters of no intervention. However, the real question one should ask is whether there are any Pareto improving policies at all. The focus is then on proving the constrained inefficiency of the competitive equilibrium which is to say that although the first best allocation cannot be achieved, a carefully designed environmental policy can still make everybody better off.

The final chapter of this Thesis has a focus on behavioral economics and environmental externalities. In particular, I focus on agents endowed with "temptation and self-control" preferences, as described first by Gul and Pesendorfer [46]. They build a 2-period decision problem where in the first period agents choose over menus of lotteries and in the second period they choose an alternative from the chosen menu. However, agents are subject to temptation: at the time of actual consumption, they suffer from urges to deviate from their "commitment" preferences which prescribes what they "should" do and instead evaluate alternatives according to their "temptation" preferences which is what they "want" to do. Importantly, even if they resist temptation, they will suffer from a self-control cost. In this chapter, I am extending this model to an economy with environmental externalities and in particular environmental pollution as a byproduct of consumption. I analyze a two- period, two-countries model where in the first period countries, represented by their decision makers, negotiate over the upper(and potentially lower) bounds of consumption and therefore over the associated level of pollution. In the second period, the consumers choose a level of consumption within the allowed range. In this context and with agents experiencing different types of temptations or random temptation, I find that it is optimal for a decision maker to commit to a singleton set avoiding in this way temptation and the cost of self-control. Allowing for shocks in the economy creates a trade-off between commitment and flexibility to adjust to shocks. In this case, the optimal policy for a decision maker will depend on the range of parameters. I find that for a relatively small shock, full commitment is favored at the expense of no adjustment to shocks. Instead, for a relatively big shock, uncertainty becomes too important to ignore and some degree of flexibility is optimal at the expense of a self-control cost. Further research will focus on the design of Pareto improving policies in this context and on extending the model to allow for strategic interactions between the countries.

Chapter 2

Uncertainty, Extreme Outcomes and Climate Change: a critique

2.1 Introduction

The issue of increasing greenhouse gas emissions and the potential impacts on welfare has been the subject of a growing literature which aims to join climate to economy, predict future temperature change and offer an insight regarding the optimal climate change policy: when, where and by how much to abate emissions. Most economic analyses of climate change policy have five elements: projections of CO_{2e} under a " business usual', projections of the average temperature change likely to result from higher CO_{2e} concentrations, projections of lost GDP and consumption resulting from higher temperatures, estimates of abatement cost of GHG emissions and assumptions about social utility and other ethical parameters. This is essentially the approach of Nordhaus [71], Stern [91], Hope [54] and others that evaluate abatement policies using "Integrated Assessment Models" (IAMs). Surprisingly, some of these models conclude - in direct contradiction of the urgency expressed in the scientific literature- that stringent abatement is both economically unsound and unnecessary.

The scope of this paper is to show how the deep structural uncertainties inherent in climate change economics can lead to very different implications for the optimal climate change policy. In Pindyck [81], the author concludes, that " given the current state of knowledge regarding warming and its impact, my results do not support the immediate adoption of stringent abatement policy." My analysis builds up on his work and aims to show how different assumptions about the utility function, the damage function and the distributions of the unknown parameters can support the immediate adoption of an abatement policy. Following Pindyck, I estimate a simple measure of the "willingness to pay": the fraction of consumption, $w^*(\tau)$ that society would be willing to sacrifice today and throughout the future to ensure that temperature change at some horizon H would be limited to τ. The value of $w^*(\tau)$ will depend on the distributions of temperature change and the economic impact as well as the utility function assumed and the values for the ethical parameters. It should be noted here that I am not considering whether the fraction of concumption sacrificed is enough to keep temperature change below τ but instead I focus entirely on how the inherent uncertainties about temperature change and economic impact would affect the WTP.

I use the distribution for temperature change and the economic impact provided by Pindyck [81] which is based on information from the IPCC (2007) and recent IAMs. As Weitzman [101] has shown, WTP becomes infinite if one uses a fat tailed distribution, therefore I follow Pindyck in using a thin tailed distribution for temperature change and economic impact. Unlike existing IAMs , he also assumes that temperature change affects the growth rate of GDP and consumption rather than the level. Although justified on theoritical and empirical grounds as Dell and Jones [30] have shown, I argue that the way he translates the economic impact in levels to one in growth is not so "innocent" and could be a source of the low WTP that he finds.

Pindyck uses a quadratic exponential loss function to relate temperature change and GDP. The choice of an exponential rather an inverse quadratic polynomial loss function (as the DICE damage function) allows for greater losses when ΔT is large. However, as have pointed out, there is no empirical evidence regarding the shape of the damage function, even at historical temperatures-let alone "out of the sample" forecasting of damages at temperatures beyond the historical range, which is what really matters in the context of uncertainty and extreme outcomes. In other words, there is no obvious source of support for the crucial assumption that the exponent is equal to 2. They use the well-established DICE damage function and study the effects of $\Delta T = 19°C$ on the level of GDP for increasing values of the exponent N. They show that with $N = 2$, less than half of world output is lost although this temperature change is far beyond the temperature change needed to end human life as we know it. In contrast, as N rises, half of the world output is lost at $\Delta T = 7°C$ for$N = 3$ and at $\Delta T = 4.5°C$ for $N = 4$. As N approaches infinity, the damage function approaches a vertical line, which makes sense if one accepts that there is a threshold for an abrupt word ending (or at least economy ending) discontinuity. As I want to study the effects of extreme climate change, I will adopt the above approach and set $N = 3$.

The final matter of concern is the choice of the utility function of the individual. Pindyck, as well as most of the IAMs, uses a CRRA "multiplicative " functional form utility which implicitly assumes perfect substitutability between material consumption and environmental amenities. However, as Tol et al [38] have pointed out, the individual derives utility from environmental amenities, not necessarily translated to market consumption. As Weitzman [101] has pointed out, the choice of an additive rather than a multiplicative form for the utility function accounts for this fact and leads to significantly different implications about climate change policy when consumption and temperature change is high, with the additive form resulting to larger values for WTP.

Therefore, I employ an "additive" as well as a "multiplicative" form for the utility function and the corresponding damage function applied to the level of consumption rather than growth while allowing for an exponent of $N = 2$ and $N = 3$. The first immediate results are higher levels of damages for large temperature change that could represent extreme climate change. These results are along the same lines with the results of Ackerman [1] and Weitzman [101] who emphasize the importance of uncertainty regarding the shape of the damage function and the climate sensitivity parameter for the inclusion of potential catastrophes in IAMs and the justification of an immediate and stringent abatement policy. Given the current " state of knowledge" Pindyck obtains estimates for the WTP which are generally below 3% even for τ around to $2-3°C$. As he states, this is because there is limited weight in the tails of the calibrated distributions for ΔT and the growth rate impact. Instead, the specifications of the model I consider, lead to significantly higher estimations for the WTP and in some extreme cases to a value close to 1. The results are even stronger in the case of a larger excepted temperature change where even a quadratic exponential loss function applied to the level of consumption gives a higher WTP than the benchmark specification of Pindyck. Although Dell, Jones and Olken [30] have shown that higher temperatures reduce GDP growth rates rather than levels, the previous results cast doubts as to whether the specification of the model where temperature change affects the growth rather than the level of consumption brings robust estimates for the WTP. However, as it is hard to argue which is the "right" functional form, the discussion is aimed to point out that a seemingly arcane theoretical distinction between a "multiplicative" and an "additive" functional form can have very different implications for the optimal climate change policy.

The rest of the paper is organized as follows. Section 2 is dedicated to the methodology followed regarding the treatment of temperature change and its distribution, and the functional forms for the damage and utility functions . In Section 3, I describe the methodology followed for the estimation of WTP as well as the differences in the estimations resulting from the different specifications of the model. Section 4 concludes with the policy implications of my analysis.

2.2 Background and Methodology

Most economic analyses of climate change policy have five elements: projections of CO_{2e} under a " business usual", projections of the average temperature change likely to result from higher CO_{2e} concentrations, projections of lost GDP and consumption resulting from higher temperatures, estimates of abatement cost of GHG emissions and assumptions about social utility and other ethical parameters. In this section, I offer a descriptive analysis of the above. As Pindyck is the benchmark model for my analysis, there is an analytical description of his methodology regarding temperature change and economic impart as well as the possible issues that can arise.

2.2.1 Temperature Change

Following Pindyck [81], I use the estimates of the IPCC (2007a) for "climate sensitivity": the temperature change that would result from an anthropomorphic doubling of CO2e concentration. Climate sensitivity is then used as a proxy of temperature change a century from now, given the IPCC's consensus prediction of a doubling (relative to the preindustrial level) of CO2e concentration by the end of the century. According to the 22 studies that the IPCC surveyed, it is found that temperature change has an expected value of 2.5° to $3.0°C$. I use the estimates of Pindyck [81] that with 17% probability, a doubling of CO2e would lead to a mean temperature increase of $4.5°C$ or more, with 5% probability to a temperature increase of 7.0 or more and with 1% probability to a temperature increase of $10.0°C$ or more. The 1% and 5% tails of the distribution for ΔT clearly represent extreme outcomes as temperature changes of such a magnitude are outside the range human experience.

I assume that temperature change follows a displaced gamma distribution and I fit it to the above summary numbers. Let θ be the displacement parameter, the distribution is given by :

$$f(x;r,\lambda,\theta) = \frac{\lambda^r}{\Gamma(r)}(x-\theta)^{r-1}e^{-\lambda(x-\theta)}, \qquad x \geq \theta \tag{2.1}$$

$$F(x;r,\lambda,\theta) = 1 - \frac{\Gamma(r,(x-\theta)\lambda)}{\Gamma(r)} \tag{2.2}$$

where

$$\Gamma(r) = \int_0^{\infty} s^{r-1} e^{-s} ds \tag{2.3}$$

is the Gamma function, r is the shape parameter and λ the inverse scale parameter.

The moment generating function for this distribution is:

$$M_{x(t)} = E\left(e^{tx}\right) = \left(\frac{\lambda}{\lambda - t}\right)^{r} e^{t\theta} \tag{2.4}$$

Hence, the mean, the variance and the skewness (around the mean) are given by $E(x) = \frac{r}{\lambda} + \theta$, $V(x) = \frac{r}{\lambda^2}$ and $S(x) = \frac{2r}{\lambda^3}$ respectively.

The next step is to fit $f(x; r, \lambda, \theta)$ to a mean of $3.0°C$ and the 1% and 5% points at $7.0°$ and $10.0°C$ respectively yields $r = 3.8$, $\lambda = 0.92$ and $\theta = -1.13$, with $V = 4.49$ and $S = 9.76$. Note that this distribution implies that there is a small probability of 2.9% that a doubling of CO2e concentration will lead to a reduction in temperature change which is consistent with several of the scientific studies. These are the estimates given by Pindyck [81]. As in Pindyck, I assume that the fitted distribution for T applies to a 100-year horizon H and that $\Delta T_t \rightarrow 2\Delta T_H$ as t gets large. This implies that the trajectory for temperature change is given by

$$\Delta T_t = 2\Delta T_H (1 - 0.5^{\frac{t}{H}}) \tag{2.5}$$

Therefore, if $\Delta T_H = 5°C$,ΔT reaches $2.93°C$ after 50 years, $5°C$ after 100 years and then gradually approaches $10°C$ as t gets large.

2.2.2 Economic Impact and Choice of Functional Forms

It has become common in the literature to assume that temperature change and GDP are associated through a loss function $L(\Delta T_t)$, with $L(0) = 1$ and $L^{'} < 0$ so that GDP at some horizon H is $L(\Delta T_H) GDP_H$, where GDP_H is GDP in the absence of global warming. Many of the IAMs use a simple power law or an inverse quadratic loss function as in the DICE model of Nordhaus [71] while Weitzman [102] argues for an exponential loss function which allows for greater losses when temperature change is large. On the other hand, there are reasons to believe that temperature change would affect the growth rate of GDP rather than just the level. One could think of irreversible and permanent effects of climate change as for example the destruction of ecosystems from erosion and flooding, extinction of species and deaths from health effects and weather extremes. Second, climate change leads to a reduction of resources available for R&D

and capital investment and therefore reduces growth as it has a negative impact on the level of GDP and also because of the resources that need to be allocated to counter the floods, droughts, sickness, etc resulting from higher temperatures. Third, there is the effect of climate change on savings. As Frankhauser et al. [38] point out, in a world with perfect foresight we can expect forward- looking agents to change their savings behavior in anticipation of future climate change. This, too, will affect the accumulation of capital and hence growth and future GDP. Finally, there is also empirical support for a growth rate effect. Dell, Jones and Olken [30] have shown that higher temperatures reduce GDP growth rates rather than levels. The impact they estimate is large-a decrease of 1.1 percentage point of growth for each $1°C$ but significant only for poorer countries. In the benchmark model of my analysis, Pindyck assumes that in the absence of warming, real GDP and consumption would grow at a constant rate g_0 but warming will reduce this rate :

$$g_t = g_0 - \gamma \Delta T_t \tag{2.6}$$

This simple linear relationship was estimated by Dell, Jones and Olken (2008), fits the data well and can be viewed as at least a first approximation to a more complex function. Assuming that the loss function applied to the levels has the following functional form

$$L(\Delta T_t) = e^{-\beta(\Delta T_t)^2} \tag{2.7}$$

Pindyck uses information from a number of IAMs to obtain a distribution for β and then translate this into a distribution for γ. To do this translation, he uses the trajectory for GDP and consumption implied by equation (2.5) for a temperature change-impact combination projected to occur at horizon H, so that the growth rate is given by

$$g_t = g_0 - \gamma 2 \Delta T_H (1 - 0.5^{t/H}) \tag{2.8}$$

Normalizing initial consumption at 1, consumption at any time t is given by the following expression

$$C_t = e^{\int_0^t g(s)ds} \tag{2.9}$$

By substituting for the growth rate according to (2.8) and solving for the integral, (2.9) becomes

$$C_t = e^{\int_0^t g(s)ds} = e^{(g_0 - 2\gamma\Delta T_H)t - \frac{2\gamma H \Delta T_H}{\ln 0.5}(1-0.5^{t/H})} \tag{2.10}$$

Noting that consumption at any time t should be the same whether I assume that temperature change affects the levels or the growth rate of GDP/consumption, γ can be obtained by equating the expressions of C_H implied by (2.7) and (2.8):

$$e^{g_0 H - \beta(\Delta T)^2} = e^{(g_0 - 2\gamma\Delta T_H)H - \frac{2\gamma H \Delta T_H}{\ln 0.5}0.5} \tag{2.11}$$

so that

$$\gamma = 1.79\beta\Delta T_H/H \tag{2.12}$$

The next step is to estimate β and hence γ. Although there is no knowledge regarding the impact of extreme temperature change in the range of $7°C$ or above, there is a consensus regarding the most likely range of economic impacts corresponding to the most likely range of temperature change. Based on its own survey of impact estimates from four IAMs, the IPCC(2007b) concludes that global mean losses could be 1-5% fo GDP for 4.0C of warming.[1] In addition, Dietz and Stern [32] provide a graphical summary of damage estimates from several IAMs, which yield a range of 0.5% to 2% of lost GDP for $\Delta T = 3.0°C$ and 1% to 8% of lost GDP for $\Delta T = 5.0°C$. Using the IPCC range and assuming that it applies to a 66% confidence interval, I take the mean loss for $\Delta T = 4.0°C$ to be 3% of GDP and the 17% and 83% confidence points to be 1% and 5% of GDP respectively. From these 3 numbers, using the exponential loss function we can find the mean, the 17% and 83% values for β which are $\bar{\beta} = 0.0019037$, $\beta_1 = 0.000628$ and $\beta_2 = 0.0032$ respectively. Pidyck translates these values of β to values of γ through (2.12) and gets $\bar{\gamma} = 0.0001367$, $\gamma_1 = 0.000045$ and $\gamma_2 = 0.00023$. Assuming that the distributions of ΔT and γ are independent given that they are governed by completely different physical economic/processes, he then fits a 3- parameter displaced gamma distribution for γ using the above numbers and he gets $r_\gamma = 4.5$, $\lambda_\gamma = 21.341$ and $\theta_\gamma = -0.0000746$.

A closer look to the assumptions and methods followed above reveals some non-trivial weaknesses that ought to be discussed. The first issue is the choice of the damage function. As noted in the introduction, there is no obvious source of support for the crucial assumption of an exponential quadratic loss function as presented in (2.7). where the exponent N is set equal to 2.[2] As Ackerman et al. [1] have pointed out, the exponent N

[1]The IAMs surveyed by he IPCC include Hope [53], Mendelsohn et at [64], Nordhaus and Boyer [74] and Tol [96].

[2]For an exponential loss function, $L(\Delta T_t) = e^{-\beta(\Delta T_t)^N}$, the exponent N measures the speed with which damages increase as temperature rises.

measures the speed with with which damages increase as temperature rises. Choosing a larger ("closer to infinity") N means moving closer to the view that complete catastrophe sets in at some finite temperature threshold. Choosing a smaller N means emphasizing the gradual rise of damages rather than the risk of discontinuous catastrophic change. As I am interested in the effects of uncertainty and extreme outcomes on the willingness to pay, I am going to use $N = 3$ as a specification of my model and compare it to the benchmark model with $N = 2$. The first obvious effect of using a larger value for N is the increase in the loss of GDP for high temperatures. For example, for $N = 3$ and an exponential loss function, I get $\bar{\beta} = 0.0006345$ and so for $\Delta T = 10°C$ (which is outside the range of human experience), almost half of the world output would be destroyed while $N = 2$ would lead to a destruction of only around 20% of world output. However, the difference is trivial for small temperature changes so for example for $\Delta T = 3.0°C$, the loss is 1.7% and 1.69% of GDP for $N = 2$ and $N = 3$ respectively.

Even if one chooses an exponent high enough to accommodate for the losses of extreme temperature change, there is an even more important source of inefficiency in the estimation of the economic impact of climate change on welfare: the functional form of the loss function, whether "multiplicative" or "additive", will have very different implications for the magnitude of the impact of temperature change. It has become common in the literature of climate change and IAM's to assume a multiplicative functional form for the loss function either with rational bounding or with exponential bounding given by

$$L_{MR}(\Delta T_t) = \frac{1}{1 + a_M \Delta T_t^2} \tag{2.13}$$

and

$$L_{ME}(\Delta T_t) = e^{-\beta(\Delta T_t)^2} \tag{2.14}$$

respectively where the latter form gives larger losses for high temperature change.[3] Then, a CRRA utility function corresponding to the first multiplicative damage function is of the form

$$U_{MR}(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)}(1 + a_M \Delta T_t^2)^{\eta - 1}}{1 - \eta} \tag{2.15}$$

and for the second

[3] At it will be shown later, WTP is higher in the case of multiplicative damage function with exponential bounding than with rational bounding.

$$U_{ME}(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)} e^{-\beta \Delta T_t^2 (1-\eta)}}{1-\eta} \tag{2.16}$$

where c_t^* is consumption in the absence of temperature change and η is the index of relative risk aversion.[4]

The above functional form assumes that there is strong substitutability between its two attributes, consumption and temperature change. It best fits situations where the main economic impact of temperature change is in material consumption or consumption of market goods. However, as Fankhauser and Tol [38] have pointed out, the individual can derive utility from non market goods, as for example environmental amenities. This idea is embodied in what is so called an "existence" value for environment. Hence, climate change which, in the model considered, is represented by a temperature change, has market impacts which go through the increase of production cost and affect the growth rate of the economy, but also non-market impacts such as the effect on recreational and environmental assets, health and biodiversity. In this account,Weitzman [102] shows that using a "additive" rather than a "multiplicative" functional form for the loss function and the corresponding utility function takes into account non-market impacts of climate change. He suggests the following "additive" loss function

and the implied "additive" utility function

$$L_A(\Delta T_t) = \frac{1}{1 + a_A c_t^* \Delta T_t^2} \tag{2.17}$$

$$U_A(c_t^*, \Delta T_t) = \frac{c_t^{*(1-\eta)} (1 + a_A c_t^* \Delta T_t^2)^{\eta - 1}}{1-\eta} \tag{2.18}$$

where it is implicitly assumed that there is no substitutability between market and non-market goods and consumption. Although it is hard to argue which is the "right" functional form, the discussion is aimed to point out that a seemingly arcane theoretical distinction between a "multiplicative" and an "additive" functional form can have very different implications for the optimal climate change policy. It will be shown later that, especially for high temperature change, willingness to pay to avoid climate change is much higher under an "additive" utility function.

The same discussion applies for the choice of function that relates growth to temperature change. Pindyck assumes a simple linear functional form, the same that Dell

[4] It should be noted that $U(c_t^*, \Delta T_t)$ represents U only as a reduced form in c_t^* and ΔT_t. The indirect pathway via which temperature change affects welfare is through reducing actual consumption as represented by the loss function $c_t = L(\Delta T_t)c_t^*$ so that $V(L(\Delta T_t)c_t^*) = U(c_t^*, \Delta T_t)$

and Jones use in their analysis. Although it is a good approximation of a possibly more complicated relationship and fits the data well, it does not account for the effects of large temperature change. First, Pindyck argues that he uses an exponential loss function in order to allow for larger losses for high temperature change in the levels of consumption, which he then translates into an effect on the growth level. However, the choice of the loss function is irrelevant for the estimation of γ: only the estimated loss from a range of likely temperature change scenarios is needed for the estimation of the effect on growth. Therefore, the choice of the growth function becomes even more important as it will determine the magnitude of the impact of high temperature change and extreme events. To make this point clear, assume that $g_0 = 0.02$,$\gamma = 0.\bar{0}001367$, and $\Delta T_H = 20.0C^{\circ}$. Equation (2.8) then implies a decrease of 0.0003 in the growth rate which is tiny for a temperature change which is well beyond the temperature needed to cause the end of human life as we know it. It comes as no surprise then that the benchmark model concludes that even if people face the risk of an extreme climate change event that would end human life as we know it, willingness to pay for abatement technology would still be very low.

2.3 Willigness to Pay

The preferences of an agent are described by a CRRA utility function:

$$V\left(c_t\right) = \frac{c_t^{1-\eta}}{1-\eta} \tag{2.19}$$

where η is the index of relative risk aversion. As discussed earlier, $U(c_t^*, \Delta T_t)$ represents utility as a reduced form in c^* and ΔT so that

$$V(L(\Delta T_t)c_t^*) = U(c_t^*, \Delta T_t)$$

Time is continuous and we normalize population to be constant at N=1. Then, social welfare is:

$$W = E_0 \int_0^{\infty} U(c_t^*, \Delta T_t)\, e^{-\delta t} dt \tag{2.20}$$

where δ is the pure time preference rate and E_0 denotes the expectation at $t = 0$ over the distribution of the unknown parameters.[5] I assume that there is just one individual

[5] As discussed earlier, I have assumed a displaced gamma distribution for temperature change and for the parameter γ.

at each point in time (or a group of identical individuals). Alternatively, one could think of the problem as one of a representative consumer that lives for infinite time. That is to say that the lifetime well-being of a person is constructed in the same way as intergenerational well-being is constructed, which means that even if a person lives for many periods, she is considered to be a distinct self at each point in time. However, one could argue here that the decision about how much an individual would save for his children involves quite different ethics than the ones involved in the decision about how one should spread his consumption across time.

In the benchmark model with a linear function describing the effect of temperature change on growth, social welfare is given by:

$$W = \frac{1}{1-\eta} E_0 \int_0^{\infty} e^{-\rho t + \omega(1-0.5^{t/H})} dt \tag{2.21}$$

where $\rho = (\eta - 1)(g_0 - 2\gamma\Delta T_H) + \delta$ and $\omega = \frac{2(\eta-1)\gamma\Delta T_H}{\ln(0.5)}$

In order to study the implications of different functional forms for the loss and utility functions, I consider three different specifications for my model: a multiplicative damage function with exponential bounding, one with rational and an additive one. Taking into account the relevant functional forms for utility as given in (2.15), (2.16),(2.18) and noting that consumption in the absence of temperature change at time t is given by $c_t^* = e^{g_0 t}$, social welfare is given by

$$W_{ME} = \frac{1}{1-\eta} E_0 \int_0^{\infty} e^{-\delta t - (\eta-1)(g_0 t - 4\beta\Delta T_H^2 (1-0.5^{t/H})^2)} dt \tag{2.22}$$

$$W_{MR} = \frac{1}{1-\eta} E_0 \int_0^{\infty} e^{-\delta t - g_0(\eta-1)t} (1 + 4\alpha_M \Delta T_H^2 (1-0.5^{t/H})^2)^{\eta-1} dt \tag{2.23}$$

$$W_A = \frac{1}{1-\eta} E_0 \int_0^{\infty} e^{-\delta t - g_0(\eta-1)t} (1 + 4e^{g_0 t}\alpha_A \Delta T_H^2 (1-0.5^{t/H})^2)^{\eta-1} dt \tag{2.24}$$

respectively.

In these three specifications, temperature change does not the enter the growth function of consumption. Instead, temperature change and consumption are associated through a loss function $L(\Delta T_t)$, with $L(0) = 1$ and $L^{'} < 0$ so that consumption at some horizon H is $L(\Delta T_H)\, c_H^*$, where c_H^* is consumption in the absence of global warming.

Note, that in order to avoid integrals that blow up, WTP must be based on some finite horizon, which I set to be N=500 years.

Suppose society sacrifices a fraction $w(\tau)$ of present and future consumption to ensure that any increase in temperature at a specific horizon H, is limited to an amount τ. This fraction is the measure of the willingness to pay. In this case, social welfare at $t = 0$ would be :

$$\tilde{W}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\tilde{\rho}t + \tilde{\omega}(1-0.5^{t/H})} dt \tag{2.25}$$

$$\tilde{W}_{ME}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - (\eta-1)(g_0 t - 4\beta \Delta T_H^2 (1-0.5^{t/H})^2)} dt \tag{2.26}$$

$$\tilde{W}_{MR}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4\alpha_M \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{2.27}$$

$$\tilde{W}_{A}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} E_{0,\tau} \int_0^\infty e^{-\delta t - g_0(\eta-1)t} (1 + 4e^{g_0 t} \alpha_A \Delta T_H^2 (1 - 0.5^{t/H})^2)^{\eta-1} dt \tag{2.28}$$

where $E_{0,\tau}$ denotes the expectation at $t = 0$ over the distributions of ΔT_H and γ conditional on $\Delta T_H \leq \tau$.[6] (I use $\tilde{\omega}$ and $\tilde{\rho}$ to denote that they are both functions of random variables.) If on the other hand, no action is taken to limit warming, ΔT_H is unconstrained and social welfare is given by (2.21), (2.22), (2.23) and (2.24), Willingness to pay to ensure that $\Delta T_H \leq \tau$ is the value $w^*(\tau)$ that equates $\tilde{W}(\tau)$ and W, social welfare without any action taken, for all the specifications of the model. By constraining ΔT_H to be under some threshold τ, damages from temperature change will also be constrained and a lower bound for consumption will be guaranteed. If no action is taken, the distribution temperature change is unconstrained and so are the damages and lost consumption. WTP is the fraction of consumption you could sacrifice today and in the future that makes you exactly indifferent between taking and not taking any action.

Given the distributions $f(\Delta T)$ and $h(\gamma)$ for ΔT and γ respectively, denote by $M_\tau(t)$, $M_\infty(t)$ the constrained and unconstrained time- t expectations for the benchmark model:

[6] Note that I have assumed a displaced gamma distribution for temperature change and the parameter γ as given in (2.12).

$$M_{\tau}(t) = \frac{1}{F(\tau)} \int_{\theta_{\tau}}^{\tau} \int_{\theta_{\gamma}}^{\infty} e^{-\tilde{\rho} t + \tilde{\omega}(1-0.5^{t/H})} f(\Delta T) g(\gamma) d\Delta T d\gamma \tag{2.29}$$

$$M_{\infty}(t) = \int_{\theta_{\tau}}^{\infty} \int_{\theta_{\gamma}}^{\infty} e^{-\tilde{\rho} t + \tilde{\omega}(1-0.5^{t/H})} f(\Delta T) g(\gamma) d\Delta T d\gamma \tag{2.30}$$

where θ_{τ} and θ_{γ} are lower limits on the distribution for ΔT and γ and $F(\tau) = \int_{\theta_{\tau}}^{\tau} f(\Delta T) d\Delta T$. The constrained expectation refers to the case when action is taken to limit temperature change and the unconstrained when no such action is taken. Therefore, social welfare in the case of action and no action taken, as given in (2.21) and (2.25), becomes

$$\tilde{W}(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} \int_{0}^{N} M_{\tau}(t) dt = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} G_{\tau} \tag{2.31}$$

$$W = \frac{1}{1-\eta} \int_{0}^{N} M_{\infty}(t) dt = \frac{1}{1-\eta} G_{\infty} \tag{2.32}$$

where $G_{\tau} = \int_{0}^{N} M_{\tau}(t) dt$ and $G_{\infty} = \int_{0}^{N} M_{\infty}(t) dt$

FInally, setting $\tilde{W}(\tau)$ equal to W, WTP is given by :

$$w^{*}(\tau) = 1 - [G_{\infty}/G_{\tau}]^{1/1-\eta} \tag{2.33}$$

Following the same method, I can find the WTP for the three specifications of my model, namely $\tilde{w}^{*}_{ME}(\tau)$, $\tilde{w}^{*}_{MR}(\tau)$ and $\tilde{w}^{*}_{A}(\tau)$.

2.3.1 Parameter Values

Finally, one should discuss the values assumed for the "ethical" parameters, namely the rate of time preference (or the utility discount rate) δ and the consumption discount rate R. A good starting point is the so-called Ramsey rule

$$R_t = \delta + \eta g_t \tag{2.34}$$

where η is the index of relative risk aversion and g_t is the growth rate of consumption. The first component of, δ, implies discounting of future utility while the second implies discounting the value of future consumption simply because we will be richer in the future and the rich gain less welfare than the poor for a given quantity of money.

There are two approaches for the choice of the consumption discount rate. The prescriptive approach sets δ and η based on ethical views and then calculates R according to the Ramsey rule. In contrast, the descriptive approach sets R based on descriptions of the financial market interest rates and then calculates δ and η. Descriptivists are often flexible on the specific values of δ and η as long as they combine to give the desired level of R. As Dasgupta argues, descriptivism "is an interesting democratic work in that the values are generated by people's behavior as they go out in their daily lives. However, one should think here who are these people whose behavior we observe ". The market interest rates describe the behavior of current individuals only. One could ask the question why the interest of future generations should be determined by consulting the preferences of present generation (Baum, [8]), an approach which is often called "dictatorship of the present". Given that the decisions of the present generation will have a long run impact that will affect the welfare of the future generations, excluding future humans is violating the basic principle that those who are affected by a decision should have input in the decision-making process. One should also note here that, due to externalities and the absence of a market for environmental amenities, the social discount rate could be substantially different from the private rate of return. Finally, one should take into account the existence of multiple market rates so that analysts cannot consider any single market rate to describe how society discounts.

Hence, I stick with the prescriptive approach and I set $\delta = 0$ on the grounds that even though most people would value a benefit today more highly than a year from now, there is no reason why society should impose those preferences on the wellbeing of our grandchildren. Cline [23] also sets $\delta = 0$ while Stern sets $\delta = 0.01$ to account for the probability of extinction. However, they both find that stringent abatement policy is necessary. In contrast, Nordhaus [71] sets $\delta = 0.03$ and concludes that aggressive abatement is not optimal. Pindyck also assumes $\delta = 0$ and the consumption discount rate is given by

$$R_t = \delta + \eta g_t = \delta + \eta g_0 - 2\eta\gamma\Delta T_H(1 - 0.5^{\frac{t}{H}}) \tag{2.35}$$

Note that in this specification, R is endogenous: it depends on the growth rate of consumption g_t which is function of temperature change ΔT_t.

As for the index of relative risk aversion, I set $\eta = 2$, which is to say that individuals are very risk averse towards inequalities in income among generations (or in different periods of their lifetime). Based upon empirical evidence, g_0 is in the range of $0.02 - 0.25$. In my estimations, I will set $g_0 = 0.015 - 0.025$.

2.3.2 No uncertainty

If the trajectory for ΔT and the impact of that trajectory on economic growth and in the loss functions were all known with certainty, social welfare with and without action would simply be:

$$W_1 = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} \int_0^N e^{-\rho_\tau t + \omega_\tau (1-0.5^{t/H})} dt \tag{2.36}$$

$$W_2 = \frac{1}{1-\eta} \int_0^N e^{-\rho_0 t - \omega t^2} dt \tag{2.37}$$

where $\rho_\tau = (\eta - 1)(g_0 - 2\gamma\tau) + \delta$ and $\omega_\tau = \frac{2(\eta-1)\gamma\tau}{\ln(0.5)}$. By setting $\Delta T_H = \tau$, I find social welfare under no uncertainly for all the specifications of my model.

I start by calculating the WTP to keep $\Delta T = 0°C$ for all time, i.e. $w^*(0)$, over a range of values for ΔT at a horizon of 100 years. Here, I am using the mean $\bar{\gamma}$ as the certainty equivalent value of γ and I set N=500, $\eta = 2$, $\delta = 0$, and $g_0 = 0.015, 0.02, 0.025$ [7]. The results for the different specifications of the model and different combinations of the parameters are shown in Table 2.1. For example, in the benchmark specification, for $\Delta T_H = 6°C$ and $g_0 = 0.02$, $w(0)^* = 0.022$ which means that society would be willing to sacrifice about 2.2% of current and future consumption to keep temperature change at $0°C$ instead of $6°C$.

The first thing to notice is that the additive specification gives a value of WTP much higher than every other specification: even for small expected temperature change, such as $\Delta T = 2°C$, WTP is in the range of 8%-14%, a range much higher than the one implied by the Pindyck model and by the alternative multiplicative specifications. Therefore, it appears that whether we assume that market goods and non-market goods are substitutable or not, has a determining effect on the optimal climate change policy. One could argue that, climate change impacts of a specific nature such as health and biodiversity qualify as non-market goods which cannot be readily translated to material consumption. In that case, perhaps an additive utility function is more appropriate from a theoretical perspective while accounting for the severity of large temperature changes and the urgency of action that the environmental scientists seem to prescribe.

A few more points worth to be noted. First, an exponent of 3 or larger in the damage function, increases WTP significantly for every specification, especially for larger changes in temperature. This result comes as no surprise: allowing for a larger exponent

[7]Similarly, I use the mean value of β, α_A and α_M for the other specifications of my model.

emphasizes the effects of extreme climate change on consumption and consequently on the willingness to pay. Comparing the case of growth effect of temperature with the one on the levels, WTP is similar for small temperature change but becomes much higher in the latter specification for larger temperature change. One should also note that the larger the expected temperature change is, the higher is the WTP. In contrast, the higher is τ, the upper bound on temperature change, the lower is the WTP. Society is more willing to pay when they are expecting worst outcomes to come and less willing, the less effective the environmental policy is. Finally, in all the specifications except from the additive, WTP increases as the initial growth rate decreases. The reason is that lowering g_0 lowers the entire trajectory for the consumption discount rate. This rate falls as ΔT increases but its starting value is $\delta + \eta g_0$. The damages from warming are initially small, making estimates of WTP highly dependent on the values for δ, η, and g_0 .

2.3.3 Uncertainty Limited to Temperature Change

The discussion above also applies to models with uncertainty. As an illustrative example, I will examine the case where there is uncertainty only with regards to temperature change: the results extent to economies with uncertainty over the economic impact too. Therefore, I assume that there is uncertainty over the trajectory of ΔT as represented by its cumulative distribution while the loss function is deterministic, with the parameters fixed at their mean values. I focus on the benchmark specification of Pindyck and the one with the additive loss function which allows for higher WTP. Table 2.2 shows $w^* (\tau)$ for several values of τ, setting $\eta = 2$, $\delta = 0$ and $g_0 = 0.015 - 0.025$. The first thing to observe is that in the benchmark specification, WTP is always below or just around 2% even for small values of τ. The values become even smaller as initial growth rate increases as the consumption discount rate becomes bigger. In contrast, in the additive specification WTP is in the range of 10-40 % and increases as initial growth increases. These are remarkably large values and one could wonder whether it is reasonable to accept that society would be willing to sacrifice around 30% of their lifetime income to prevent a temperature change of $2°C$. However, it is indicative for the discussion of how different assumptions regarding the functional forms can have to radically different policy implications.

2.4 Policy Implications and Conclusions

The policy implications of these results are stark. As Pindyck states, in order to support a stringent abatement policy, one has to find values for the WTP around 2-3%. However, following his specification, the results show that WTP is below 2% even for small values of τ. In the case of no uncertainly, WTP to avoid any temperature change increases as the expected temperature change increases but it is still only around 6% when one expects $\Delta T = 10°C$ which is far beyond the temperature change needed to end human life as we know it. Although these estimates do not support the immediate adoption of a stringent GHG abatement policy, they do not imply that no abatement at all is optimal. For example, values of the WTP close to 2% is in the range of cost estimates for compliance with the Kyoto Protocol.

I therefore examine alternative specifications of the benchmark model. Employing a multiplicative damage function applied on the level instead of the growth of consumption leads to an increase in the WTP but only for large values of expected temperature change. It becomes higher though when we allow for an exponent of 3 or larger in the damage function as it better accounts for extreme events. However, the crucial assumption in these specifications is the substitutability between market and non-market consumption. If one assumes that the main impact of temperature change is on things that cannot be easily translated to material consumption such as health and biodiversity, then an additive rather than a multiplicative utility function has to be employed. My estimates show, that in this specification WTP gets remarkably high values even if expected temperature change is relatively small or the target temperature change is relatively big.

Although one could not strongly argue which is the right specification for the model, the main purpose of this paper is to show how seemingly small differences in modeling can have very different policy implications. Deep structural uncertainties inherent in the climate change science require a careful treatment in order for the analysis to lead to robust results. Moreover, a more realistic model that accounts for parametric as well as intrinsic uncertainty in the form of exogenous shocks and risks needs to be developed, if we are to safely answer the question of whether a stringent abatement policy needs to be employed.

2.5 Tables

	Pind. Model			*Exponential Bounding*			*Rational Bounding*			*Additive*		
ΔT	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025
2	0.011	0.007	0.005	0.006	0.003	0.002	0.005	0.003	0.002	0.086	0.11	0.135
4	0.023	0.014	0.009	0.02	0.013	0.009	0.018	0.013	0.009	0.272	0.332	0.384
6	0.035	0.022	0.014	0.044	0.029	0.021	0.04	0.028	0.021	0.457	0.528	0.583
8	0.048	0.029	0.019	0.077	0.053	0.038	0.069	0.049	0.037	0.51	0.665	0.713
10	0.06	0.036	0.024	0.119	0.083	0.06	0.103	0.075	0.056	0.700	0.757	0.795

TABLE 2.1: No Uncertainty, $\tau = 0$

	Pindyck			*Additive*		
τ	g=0.015	g=0.02	g=0.025	g=0.015	g=0.02	g=0.025
0	0.019	0.011	0.007	0.311	0.376	0.429
2	0.011	0.007	0.005	0.280	0.338	0.385
4	0.006	0.004	0.002	0.194	0.234	0.268
6	0.003	0.002	0.001	0.108	0.13	0.148

Table 2.2: Uncertainty Limited to Temperature Change

Chapter 3

When financial imperfections are not the problem but the solution

3.1 Introduction

The BP Deepwater Horizon oil spill of 2010 has focused considerable attention on the potential liability and the operating conduct of big oil companies. In this case, BP's unusually "deep pockets", made full compensation to the victims feasible, drawing attention away from the glaring safety failures in both the private and the public sector. Big oil companies make an annual purchase of roughly \$500 millions of liability insurance, an amount much lower than the \$20 billion fund that BP was able to self-raise. It seems that big oil companies, the same ones that are more likely to cause a catastrophic oil spillage, do not need to buy liability insurance at all: they have a sufficiently diversified investment portfolio that allows them to self-insure for the vast majority of their potential catastrophic liability. Future spills, however, may not follow this pattern revealing the need to create incentives for the parties involved in activities with potential catastrophic environmental consequences, to take actions to prevent such events.

In this spirit, this paper suggests that, restricting the insurance opportunities of big oil companies in the asset markets creates incentives to internalize the welfare effects of such catastrophic events, leading to a Pareto improved allocation for society as a whole. I model a class of general equilibrium economies with uncertainty, where the probability of each state rather than being exogenous, depends on the level of effort exerted by one agent. Thus, the probabilities of different states are endogenously determined in equilibrium. When this "effort" is costly and not contracted upon, every equilibrium is Pareto inefficient as in all economies with imperfections, an "externality" in this case. The intuition behind this result is straightforward: when the agent decides on the optimal level of effort, he only takes into account the effects on his own welfare.

However, as his choice of effort affects the probability of each state, it also affects the expected social welfare, an effect that the agent fails to internalize. I then focus on welfare improving policies, in particular on the imposition of "participation" constraints in financial markets. I show that generically in the space of endowments, there is a Pareto-improving policy in the form of a reallocation of existing assets. The results extend to economies with aggregate uncertainty and complete markets as well as to economies with uninsurable idiosyncratic risk. This has the important implication that complete markets need not to be optimal in economies with this type of externalities. Restricting the insurance opportunities of agents is welfare improving for society as a whole as it forces the agent to "internalize" the social cost of his choice of effort by making him more vulnerable to risk and uncertainty.

It is well known that any equilibrium allocation is Pareto optimal in competitive economies with complete markets. Constrained optimality is a weaker version of optimality that takes into account the financial structure and is appropriate for the case of incomplete markets. As Stiglitz [92], Greenwald and Stiglitz [44] and Geanakoplos and Polemarchakis [41] have argued, in a numeraire asset model with incomplete markets, generically every equilibrium is constrained sub-optimal provided that there is more than one commodities and there is an upper bound in the number of individuals: reallocations of existing assets support superior allocations. Following Geanakoplos and Polemarchakis [41], Citanna, Kajii, Villanacci, [21] show that equilibria are generically constrained inefficient even without an upper bound on the number of households. They also show that perfectly anticipated lump-sum transfers in a limited number of goods are typically effective. Carvajal and Polemarchakis [16] extend these results to economies with no aggregate uncertainty and uninsurable idiosyncratic risk where the reallocation of assets works through its effects on future relative prices.

The inefficiency of incomplete markets is the benchmark for the financial innovation literature. Hart [50] was the first to construct an example of a competitive economy where the equilibrium allocation with a single asset is Pareto dominated by one without no asset at all. Therefore, contrary to what one might have thought, the introduction of a new asset does not always improve upon welfare. As Cass and Cittana [18] and Elul [35] have argued, introducing a new asset in an economy with incomplete markets can be Pareto-improving or Pareto-impairing. Therefore, the choice of which assets to introduce has important welfare effects. This result has given rise to the literature on optimal security design. Prime examples of this strand of literature include Allen and Gale [3] and [4], Chen [20] and Pesendorfer [78] where the main focus is the trade-off between profitable innovation and the cost of financial intermediation as a source of the incompleteness of the financial markets in equilibrium. Bisin [10] shows that in a environment with profit maximizing intermediaries and intermediatery costs, markets are endogenously incomplete in equilibrium. Carvajal, Rostek and Weretka [17] argue that the main force for the incompleteness of markets in equilibrium is the shape of the marginal utility function of the investors: under convexity, competition among issuers of asset-backed securities who maximize asset value, results in incomplete financial structures and inefficient risk sharing opportunities for the investors even if innovation is essentially costless.

My work touches upon the literature on moral hazard and financial innovation. In general equilibrium models with moral hazard, there is trade-off between higher insurance opportunities and incentives to exert effort when effort is unobservable: the easier it is to hedge risk, the fewer are the incentives to exert costly effort and increase the probability of being in a "good state". As a consequence, one could end up trapped in a "bad" equilibrium where insurance is expensive and the "bad" state happens with high probability. In these models, even when the individual probability of being in personal state s is independent of the effort of other agents, individuals' effort choices always affect each other via prices, "a pecuniary" effect defined by Greenwald and Stiglitz [44] as "moral hazard pecuniary externality". In addition, my model allows for externalities in the traditional sense: an agent's welfare depends on the effort exerted by one agent through its effect on the probability of the aggregate state, creating an additional source of inefficiency. However, the scope of this paper is not to solve the problem of moral hazard but rather to internalize the effect of the choice of effort on social welfare. In the moral hazard literature, the focus is on solving the principal-agent problem or in the case of multi-agent setup, on the reallocation of resources among individuals in order to solve the trade-off between incentives and risk sharing. Prime examples of this literature include Grossman and Hart [45] and Prescott and Townsend [83]. Helpman and Laffont [51] extend the moral hazard setup to a general equilibrium model with exogenously complete financial markets and non-exclusive transactions. A natural extension of this framework is to endogenize the financial design. In this direction, Lisboa [63] presents a general equilibrium model with individual and aggregate risk, moral hazard and a large number of households where insurance is supplied by a collection of profit maximizing firms that behave strategically. He shows that given the assumptions of Bentrand competition and exclusivity, an equilibrium exists and is constrained optimal. On the other hand, Braido [14] presents a general equilibrium model as a two-stage game where agents act as producers, consumers and as financial intermediaries with intermediation costs. Each individual is allowed to design a financial structure that consists of specifying securities payoffs in each state and transaction constraints that restrict the participation of some agents in some markets, while allowing for non-exclusivity. He shows that an equilibrium exists and he offers examples and reasoning as to why the equilibrium might be constrained inefficient and the markets endogenously incomplete. As he points out, apart from the financial technology and the strategic nature of financial innovation as standard sources of inefficiency, the main source would be the non-exclusivity of contracts. The inefficiency of equilibria in economies with moral hazard and non-exclusive contracts has been extensively studied by Helpman and Laffont [51], Arnott and Stiglitz [6], Bisin and Guaitoli [11] and Kahn and Mookherjee [55] ,where the main conclusion is that exclusivity is necessary for constrained efficiency of equilibria. It is important to point out the relevance of the work of Braido [14] for my model. To show how markets can be endogenously incomplete in equilibrium and how this might be optimal, he offers an example with two agents where only one of them is risk averse. This agent faces a production risk where the probability of the "good" state is increasing in the costly unobservable effort he exerts. In this setup, he shows that an incomplete financial structure, in the form of trading constraints, is Pareto superior to complete markets. My framework is more general: all agents are risk-averse and face aggregate and/or idiosyncratic risk. In my setup, the agent who exerts effort creates an "externality". Moreover, by imposing trading constraints, I am limiting the insurance opportunities of the society and it is therefore far more demanding to show that a Pareto improving policy exists.

Finally, the welfare improving policies that I analyze have been the subject of the restricted participation literature. As Polemarchakis and Siconolfi [82] point out, incomplete markets are just a special case of an asset market with restricted participation. In this setting, they prove the generic existence of competitive equilibria when agents face asymmetric linear constraints on their portfolio incomes. Cass, Siconolfi and Villanaci [19] extend the literature by accommodating a wide range of portfolio constraints, including any smooth, quasi-concave inequality constraint on households' portfolio holdings. Gori, Pireddu and Villanacci [43] focus on price-dependent borrowing restrictions. After proving the existence of equilibria, they show that equilibria associated with a sufficiently high number of strictly binding participation constraints in the financial markets can be Pareto improved upon by a local change in these constraints.

The paper is organized as follows. Section 2 starts with the basic setup of an economy with aggregate uncertainty and incomplete markets. I then present the problem of the agent and the competitive equilibrium. I show that due to the externality created by the inefficient level of effort chosen by the agent, equilibrium allocations are Pareto inefficient. I then look for Pareto-improving policies. I show that generically in the space of endowments such a policy exists. Section 3 extends my results to economies with incomplete markets and idiosyncratic risk.

3.2 Aggregate Uncertainty and Complete Markets

Consider a two-period economy and $I+1$ individuals denoted by $i = 0, ..., I$. In period 0, agents receive an endowment of the single consumption good $e_{i,0}$. In period 1, there is uncertainty regarding endowments. There are only two states of the world, which I denote by s, with $s = 1, 2$. Agents receive an endowment $e_{i,1}$ in state 1 and $e_{i,2}$ in state 2. The probabilities of each state, rather than being exogenous, depend on the actions of agent 0. In period 0, he has the choice of exerting costly "effort" $\epsilon \geq \bar{\epsilon}$, which makes state 1 more probable. As in standard models with moral hazard, effort is unobservable and agent 0 only takes into account the effects of his choice on his own welfare.

The preferences of the agents, who derive utility only from the single consumption good, are described by

$$U^i = c_{i,0} + \pi(\epsilon)u_{i,1}(c_{i,1}) + (1 - \pi(\epsilon))u_{i,2}(c_{i,2}) \tag{3.1}$$

where $\epsilon \in (\bar{\epsilon}, \infty)$ with $\bar{\epsilon}$ a lower bound for effort, $\pi(\epsilon)$ is the probability of state 1 and $(1-\pi(\epsilon))$ the probability of state 2. I assume that $\pi(\epsilon) \geq 0$ is an increasing and concave function with $\lim_{\epsilon \to \infty} \pi(\epsilon) = 1$. The state dependent utility function $u_{i,s}(c_{i,s})$ is assumed to be continuous, strictly increasing, strongly concave and to satisfy standard Inada conditions.[1][2]

Definition 1: Let $\{\hat{c}_{i,s}\}_{i=0}^{I}$ with $s = 1, 2$, solve the following maximization problem

$$\max_{\{c_{i,s}\}_{i=0}^{I}} \sum_{i=0}^{I} u_{i,s}(c_{i,s}) \quad s.t. \quad \sum_{i=0}^{I} c_{i,s} = \sum_{i=0}^{I} e_{i,s} \qquad \text{for } s = 1, 2$$

This gives us the Pareto optimal allocations state by state. Later, I will show that the same optimality conditions hold at the competitive equilibrium for any given level of effort.

Assumption 1: There exists $\epsilon \in (\bar{\epsilon}, \infty)$ such that

$$\pi'(\epsilon) = \frac{1}{u_{0,1}(\hat{c}_{0,1}) - u_{0,2}(\hat{c}_{0,2})},$$

[1] If f is differentiable on the open convex set $C \in R^n$, then it is strongly concave if and only if there exists $a > 0$ such that for every $x_1 \in C$ and every $x_2 \in C$, we have

$$f(x_2) - f(x_1) + (x_2 - x_1)^T \nabla f(x_1) \leq \frac{1}{2} a \parallel x_2 - x_1 \parallel^2$$

[2]With quasilinear preferences, it is not necessary to specify the endowments of individuals at date 0. Marginal utility is constant and equal to 1 and there is no income effect. With a more general set of preferences, computation would be more challenging and a genericitty analysis of preferences might be needed but the main line of the argument shold not change.

with $\{\hat{c}_{i,1}, \hat{c}_{i,2}\}_{i=0}^{I}$ as in Definition 1. This interiority assumption implies that agent 0 prefers state 1 to state 2 . Therefore it is optimal for him to exert effort since the probability of state 1, $\pi(\epsilon)$, is increasing in effort.

Finally, there is an Arrow security for each state: asset 1 pays one unit of the consumption good only in state 1 and asset 2 pays only in state 2. Holdings of the securities are denoted by $\theta_{i,1}$ and $\theta_{i,2}$ respectively with $\theta_{i,s} \in \mathbb{R}$. Therefore, the markets are complete in the sense that there is a security for each state. It should be noted that in my setup, effort is not contractible.

3.2.1 The problem of agent $i \geq 1$

All agents other than agent 0 have only one decision to make in period 0: they have to choose their holdings of securities, $\theta_{i,1}$ and $\theta_{i,2}$ and therefore their consumption in that period and in every state in period 1, taking the prices of the securities and the probabilities of the states as given. They face the following period 0 budget constraint

$$c_{i,0} + q_1\theta_{i,1} + q_2\theta_{i,2} \leq e_{i,0}, \tag{3.2}$$

$$c_{i,0} \geq 0 \tag{3.3}$$

and in every state in period 1

$$c_{i,1} \leq e_{i,1} + \theta_{i,1}, \tag{3.4}$$

$$c_{i,2} \leq e_{i,2} + \theta_{i,2}, \tag{3.5}$$

$$c_{i,1} \geq 0, \tag{3.6}$$

$$c_{i,2} \geq 0 \tag{3.7}$$

where q_1 and q_2 are the prices for $\theta_{i,1}$ and $\theta_{i,2}$, respectively, while the single consumption good is the numeraire.[3] The maximization problem of the agents then becomes:

$$\max_{\theta_{i,1},\theta_{i,2}} e_{i,0} - q_1\theta_{i,1} - q_2\theta_{i,2} + \pi(\epsilon)u_{i,1}(e_{i,1} + \theta_{i,1}) + (1 - \pi(\epsilon))u_{i,2}(e_{i,2} + \theta_{i,2}), \tag{3.8}$$

[3]The non-negativity constraints for consumption imply a lower bound on short sales of securities.

with the following first-order conditions:

$$-q_1 + \pi(\epsilon)\frac{\partial u_{i,1}}{\partial c_{i,1}}(e_{i,1} + \theta_{i,1}) = 0, \tag{3.9}$$

$$-q_2 + (1 - \pi(\epsilon))\frac{\partial u_{i,2}}{\partial c_{i,2}}(e_{i,2} + \theta_{i,2}) = 0. \tag{3.10}$$

3.2.2 The problem of agent $i = 0$

Agent 0 has an extra decision to make in period 0: apart from choosing his holdings of securities, he must choose his level of effort. He faces the following budget constraints:

$$c_{0,0} + q_1\theta_{0,1} + q_2\theta_{0,2} \leq e_{0,0} - \epsilon, \tag{3.11}$$

$$c_{0,1} \leq e_{0,1} + \theta_{0,1}, \tag{3.12}$$

$$c_{0,2} \leq e_{0,2} + \theta_{0,2}, \tag{3.13}$$

$$c_{0,0} \geq 0, \tag{3.14}$$

$$c_{0,1} \geq 0, \tag{3.15}$$

$$c_{0,2} \geq 0 \tag{3.16}$$

His maximisation problem becomes:

$$\max_{\theta_{0,1},\theta_{0,2},\epsilon} e_{0,0} - q_1\theta_{0,1} - q_2\theta_{0,2} - \epsilon + \pi(\epsilon)u_{0,1}(e_{0,1} + \theta_{0,1}) + (1 - \pi(\epsilon))u_{0,2}(e_{0,2} + \theta_{0,2}), \tag{3.17}$$

with the following first order conditions:[4]

$$-1 + \pi'(\epsilon)(u_{0,1}(e_{0,1} + \theta_{0,1}) - u_{0,2}(e_{0,2} + \theta_{0,2})) = 0, \tag{3.18}$$

$$-q_1 + \pi(\epsilon)\frac{\partial u_{0,1}}{\partial c_{0,1}}(e_{0,1} + \theta_{0,1}) = 0, \tag{3.19}$$

$$-q_2 + (1 - \pi(\epsilon))\frac{\partial u_{0,2}}{\partial c_{0,2}}(e_{0,2} + \theta_{0,2}) = 0. \tag{3.20}$$

[4] Assumption 1 will guarantee an interior equilibrium for the optimal level of ϵ, therefore we need not look for boundary solutions.

Condition (3.18) implies that agent 0 prefers state 1 to state 2. Therefore, he is willing to exert effort to raise the probability of state 1. However, in his choice of effort, he does not internalize the effect of a more probable state 1 on society as a whole. Since I have made no extra assumptions on aggregate endowments and social welfare in each state, all agents other than agent 0 could be worse-off or better-off in state 1. It is this feature that is reflecting the non-alignments of interest.

3.2.3 Competitive Equilibrium and Pareto Efficiency

The first-order conditions derived above, together with the market-clearing conditions for the consumption good and securities

$$\sum_{i=0}^{I} c_{i,0} = \sum_{i=0}^{I} e_{i,0} - \epsilon, \tag{3.21}$$

$$\sum_{i=0}^{I} c_{i,s} = \sum_{i=0}^{I} e_{i,s} \qquad \text{for } s = 1,2, \tag{3.22}$$

$$\sum_{i=0}^{I} \theta_{i,s} = 0 \qquad \text{for } s = 1,2, \tag{3.23}$$

give us the equilibrium prices, quantities and effort $\{\theta^*_{i,1}, \theta^*_{i,2}, c^*_{i,0}, c^*_{i,1}, c^*_{i,2}, \epsilon^*, q^*_1, q^*_2\}$.

On the other hand, a social planner would choose the allocation of consumption and the level of effort so as to maximise

$$\sum_{i=0}^{I} [c_{i,0} + \pi(\epsilon)u_{i,1}(c_{i,1}) + (1 - \pi(\epsilon)u_{i,2}(c_{i,2})], \tag{3.24}$$

subject to aggregate resource constraints:

$$\sum_{i=0}^{I} c_{i,0} = \sum_{i=0}^{I} e_{i,0} - \epsilon, \tag{3.25}$$

$$\sum_{i=0}^{I} c_{i,i} = \sum_{i=0}^{I} e_{i,1}, \tag{3.26}$$

$$\sum_{i=0}^{I} c_{i,2} = \sum_{i=0}^{I} e_{i,2}, \tag{3.27}$$

Given the quasilinerity of preferences and the above constraints, the first-order conditions that characterize Pareto efficiency are:

$$-1+\pi'(\epsilon)\left(\sum_{i=0}^{I} u_{i,1}(c_{i,1})-\sum_{i=0}^{I} u_{i,2}(c_{i,2})\right)=0 \qquad \text{for an interior solution,} \tag{3.28}$$

$$\pi(\epsilon)\frac{\partial u_{i,1}(c_1^i)}{\partial c_1^i}-\lambda_1=0 \qquad \text{for every agent } i=0,...I, \tag{3.29}$$

$$(1-\pi(\epsilon))\frac{\partial u_{i,2}(c_{i,2})}{\partial c_{i,2}}-\lambda_2=0 \qquad \text{for every agent } i=0,...I, \tag{3.30}$$

where λ_s is the Lagrange multiplier associated with the resource constraint in $s=1,2$. Solving for the multipliers, the following conditions fully characterize the equilibrium allocation and prices:

$$\pi'(\epsilon)\left(\sum_{i=0}^{I} u_{i,1}(c_{i,1})-\sum_{i=0}^{I} u_{i,2}(c_{i,2})\right)=1 \qquad \text{for an interior solution,} \tag{3.31}$$

$$\frac{\partial u_{i,1}(c_{i,1})}{\partial c_{i,1}}=\frac{\partial u_{j,1}(c_{j,1})}{\partial c_{j,1}} \qquad \text{for every agent } i \text{ and } j \text{ with } i\neq j, \tag{3.32}$$

$$\frac{\partial u_{i,2}(c_{i,2})}{\partial c_{i,2}}=\frac{\partial u_{j,2}(c_{j,2})}{\partial c_{j,2}} \qquad \text{for every agent } i \text{ and } j \text{ with } i\neq j, \tag{3.33}$$

Looking at equations (3.32) and (3.33) and comparing them with the FOC's of the agent's problem, it is easy to see that for a given level of effort, the allocations of consumption prescribed by the competitive equilibrium would be Pareto efficient in the sense that the marginal rates of substitutions would be equalized across agents in each state. However, comparing (3.18) with (3.31), it is immediate that the level of effort implied by the competitive equilibrium solution is generically not Pareto optimal: while agent 0 takes into account only the effects on his own welfare, the social planner considers the effects on social welfare when choosing the optimal level of effort.[5]

3.2.4 Constrained inefficiency and Welfare improving policies

I have shown that competitive equilibria need not be Pareto efficient. However, this does not mean that a social planner, constrained by the financial structure, would indeed be

[5]For simplicity we have shown the FOC for an interior ϵ and the Pareto inefficiency of the competitive equilibrium level. For a boundary solution with the optimal level of effort at $\bar{\epsilon}$, it is easy to see that the competitive level of effort is inefficiently high.

able to find a welfare improving policy. If such a welfare improving policy exists, then I say that the competitive equilibrium is constrained-inefficient.

I start by writing the expression for social welfare:

$$SW = -\epsilon + \pi(\epsilon)\left(\sum_{i=1}^{I} u_{i,1}(c_{i,1}) + u_{0,1}(c_{0,1})\right) + (1-\pi(\epsilon))\left(\sum_{i=1}^{I} u_{i,2}(c_{i,2}) + u_{0,2}(c_{0,2})\right), \tag{3.34}$$

The problem arises from the inefficient level of effort chosen by agent 0 when maximizing his utility. Because his effort is unobservable, I consider a perturbation in the holdings of securities of agent 0. The idea is to restrict his insurance opportunities to make him more vulnerable to the risks associated with each state. The perturbation $(d\theta_{0,1}, d\theta_{0,2})$ around the competitive equilibrium values gives us the following expression

$$\begin{aligned} dSW =& - d\epsilon + \pi'(\epsilon)\left(\sum_{i=1}^{I} u_{i,1}(c_{i,1}) + u_{0,1}(c_{0,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2}) - u_{0,2}(c_{0,2})\right) d\epsilon \\ &+ \pi(\epsilon)\left(\sum_{i=1}^{I}\left(\frac{\partial u_{i,1}(c_{i,1})}{\partial c_{i,1}} dc_{i,1}\right) + \frac{\partial u_{0,1}(c_{0,1})}{\partial c_{0,1}} dc_{0,1}\right) \\ &+ (1-\pi(\epsilon))\left(\sum_{i=1}^{I}\left(\frac{\partial u_{i,2}(c_{i,2})}{\partial c_{i,2}} dc_{i,2}\right) + \frac{\partial u_{0,2}(c_{0,2})}{\partial c_{0,2}} dc_{0,2}\right) \end{aligned} \tag{3.35}$$

The first line of equation (3.35) shows the welfare effects associated with a change in effort induced by the perturbation of asset holdings, and the second and third lines show the direct effects of a reallocation of assets. Since, the only source of inefficiency in our model is the level of effort, the terms in the second and third line vanish. Indeed using (3.18), (3.32) and (3.33), equation (3.35) can be rewritten as

$$\begin{aligned} dSW =& \pi'(\epsilon)\left(\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2})\right) d\epsilon + \pi(\epsilon)\left(\frac{\partial u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}} \sum_{i=0}^{I} dc_{i,1}\right) \\ &+ (1-\pi(\epsilon))\left(\frac{\partial u_{0,2}(\hat{c}_{0,2})}{\partial c_{0,2}} \sum_{i=0}^{I} dc_{i,2}\right) \end{aligned} \tag{3.36}$$

At equilibrium $\sum_{i=0}^{I} dc_{i,1} = 0$ and $\sum_{i=0}^{I} dc_{i,2} = 0$, so that I can finally write

$$dSW = \pi'(\epsilon)\left(\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2})\right) d\epsilon \tag{3.37}$$

where $d\epsilon$ is the change in the optimal level of effort induced by the perturbation of asset holdings in (3.18)

$$\pi''(\epsilon)(u_{0,1}(e_{0,1}+\theta_{0,1})-u_{0,2}(e_{0,2}+\theta_{0,2}))d\epsilon+\pi'(\epsilon)\left(\frac{\partial u_{0,1}}{\partial c_{0,1}}(e_{0,1}+\theta_{0,1})dc_{0,1} - \frac{\partial u_{0,2}}{\partial c_{0,2}}(e_{0,2}+\theta_{0,2})dc_{0,2}\right) = 0, \tag{3.38}$$

so that[6]

$$d\epsilon = \frac{\pi'(\epsilon)\left(\frac{\partial u_{0,2}}{\partial c_{0,2}}(e_{0,2}+\theta_{0,2})dc_{0,2} - \frac{\partial u_{0,1}(e_{0,1}+\theta_{0,1})}{\partial c_{0,1}}dc_{0,1}\right)}{\pi''(\epsilon)(u_{0,1}(e_{0,1}+\theta_{0,1}) - u_{0,2}(e_{0,2}+\theta_{0,2}))} \tag{3.39}$$

The direction of the Pareto improving policy depends on the sign of $\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2})$. Positive values imply that society, excluding agent 0, is better off in state 1 than in state 2. As a result, $d\epsilon$ must be positive: the competitive level of effort is too low and the Pareto improving policy involves inducing agent 0 to increase the level of effort chosen at equilibrium. Looking at (3.39), this can be achieved with $d\theta_{0,2} < 0$ and $d\theta_{0,1} > 0$. The idea is that in order to induce agent 0 to exert more effort in equilibrium, I would like to restrict his insurance opportunities in a way that he is better off in the state associated with higher effort, namely state 1 and worse off in state 2. In this way, due to the lack of optimal insurance, the agent would like to make state 1, in which he is now better off, more probable by choosing a higher level of effort than before. The agent is therefore "forced" to internalize the externality through considerations for his own welfare. In the case where $\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2}) < 0$, the society excluding agent 0 is better off in state 2, so that the competitive level of effort is inefficiently high: in this case, Pareto optimality prescribes $d\epsilon < 0$ which can be achieved with $d\theta_{0,2} > 0$ and $d\theta_{0,1} < 0$. The intuition is again that I would like to make agent 0 better off in the state associated with lower effort and worse off in the other state so that it is optimal for him to choose a lower level of effort than before. I should note that it can be easily shown that $\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2}) \neq 0$ generically in the space of endowments so that there is almost always a Pareto improving policy. The proof is shown in the appendix.

[6] Note that Assumption 1 implies that in equilibrium, $(u_{0,1}(e_{0,1} + \theta_{0,1}) - u_{0,2}(e_{0,2} + \theta_{0,2})) > 0$.

Remark 1: I have considered a policy intervention in the form of a reallocation of assets for agent 0. However, the same analysis applies if one wants to consider imposing binding participation constraints in the financial markets. In that case, I would start with constraints or upper and lower bounds on sales and purchases of securities, exactly at the levels of the original competitive equilibrium. Then, by making these constraints or bounds stricter therefore binding for agent 0, I can study the welfare effects of limiting the insurance opportunities and inducing a new level effort at equilibrium. However in this case it is important to check for potential non-convexities.

Remark 2: For simplicity, I have assumed there are only two states in period 1. However, it is easy to show that the results extend to economies with any finite number of states. The model and the proofs for such an economy are presented in the Appendix B.

The above analysis suggests that it is possible to correct a failure in the market by creating one more: by restricting the access of agent 0 in the insurance market and therefore limiting the insurance opportunities for the rest of the agents too, I get equilibrium allocations which are Pareto superior to the ones without any constraints. Therefore, in the particular structure of our model, complete markets are not enough for Pareto efficiency. The traditional theory in the restricted participation literature seems to conclude that removing or relaxing these constraints is welfare improving. Instead, in my setup, the imposition of such restrictions and the resulting insurance inefficiency seem to be welfare improving for the society as a whole.

3.3 Idiosyncratic risk and Incomplete markets

As an alternative framework and in order to show that my results extend to economies with idiosyncratic risk, I will study a model of incomplete markets where agents are subject to idiosyncratic shocks. Individuals are of different types $i = 0, ..., I$ and within each type there is a continuum of individuals of mass 1. Individuals of different types differ in their period 1 preferences and in their endowments but they face the same idiosyncratic shocks. For simplicity, I assume that there are only 3 personal states denoted by s, with $s = 1, 2, 3$. In $s = 1$ there is no shock in the endowment of the consumption good while in $s = 2, 3$ there is a positive and negative shock respectively of size z. In period 1, a fraction $\pi(\epsilon)$ of the individuals shall find themselves in $s = 1$ while an equal fraction of $\frac{1}{2}(1 - \pi(\epsilon))$ shall find themselves in $s = 2, 3$ respectively. However, there is no aggregate uncertainty: the aggregate endowment of the economy in period 1 is $\sum_{i=0}^{I} \bar{e}^i$ where $\bar{e}^i$ is the endowment of an individual of type i when there is no shock.[7]

Once again, the probabilities of each personal state depend on the aggregate effort agents of type 0 will choose to exert. In particular, while the expected value of endowment remains unchanged with effort, higher effort increases the probability of no shock while it decreases the probability of a positive or negative shock. Therefore effort has an effect on the variance of the distribution of shocks but not on the expected value: a risk averse agent would prefer a less "variant"distribution and this is why he would decide to exert effort.

Finally, there is only a riskless asset that can be traded: it pays one unit of the consumption good at date 1. Holdings of the asset are b^i and its price is q.

3.3.1 Competitive Equilibrium and Constrained Inefficiency

Once again, all agents other than agent 0, have to choose their holdings of the riskless bond and therefore the consumption in period 0 and in every personal state in period 1. The maximization problem an agent of type i faces is

$$\begin{aligned}\max_{b^i} e_0^i - qb^i + \pi(\bar{\epsilon}^0)u^i(\bar{e}^i + b^i) \\ + \frac{1}{2}(1-\pi(\bar{\epsilon}^0))u^i(\bar{e}^i + z + b^i) + \frac{1}{2}(1-\pi(\bar{\epsilon}^0))u^i(\bar{e}^i - z + b^i)\end{aligned} \tag{3.40}$$

where $\bar{e}^i$ is the endowment of individual in $s = 1$ when there is no shock and z is the shock. The probability of each state depends on the aggregate level of effort exerted by agents of type 0, denoted with $\bar{\epsilon}^0$. Taking the price of the riskless bond and the probabilities of the personal states as given, they choose b^i according to

$$\begin{aligned}q =& \pi(\bar{\epsilon}^0)\frac{\partial u^i}{\partial c^i}(e^i + b^i) \\ &+ \frac{1}{2}(1-\pi(\bar{\epsilon}^0))\left(\frac{\partial u^i}{\partial c^i}(e^i + z + b^i) + \frac{\partial u^i}{\partial c^i}(e^i - z + b^i)\right)\end{aligned} \tag{3.41}$$

[7] Let the endowment of an individual of type i in states 1,2 and 3 be given by $\bar{e}^i$, $\bar{e}^i + z$ and $\bar{e}^i - z$ respectively. There is no aggregate risk and the aggregate endowment of the economy is given by

$$\sum_{i=0}^{I}\left(\pi(\epsilon)\bar{e}^i + \frac{1}{2}(1-\pi(\epsilon))(\bar{e}^i + z) + \frac{1}{2}(1-\pi(\epsilon))(\bar{e}^i - z)\right) = \sum_{i=0}^{I}\bar{e}^i$$

On the other hand, agent j of type 0 has to solve

$$\max_{b_j^0,\epsilon_j^0} e_{j0}^0 - qb_j^0 - \epsilon_j^0 + \pi(\epsilon_j^0)u_j^0(\bar{e}_j^0 + b_j^0) + \frac{1}{2}(1-\pi(\epsilon_j^0))u_j^0(\bar{e}_j^0 + z + b_j^0) + \frac{1}{2}(1-\pi(\epsilon_j^0))u_j^0(\bar{e}_j^0 - z + b_j^0) \tag{3.42}$$

It is imporant to note that although I have assumed that the probability of each state depends on the aggregate effort exerted by all agents of type 0, when an agent j of type 0 solves his maximization problem, he sees the probability of each state as depending only on his own level of effort. In equilibrium, as there is a continuum of type 0 agents of mass 1, it is true that $\epsilon_j^0 = \bar{\epsilon}^0$.

Now, the FOC are[8]

$$1 = \pi'(\epsilon_j^0)\left((u_j^0(e_j^0 + b_j^0) - \frac{1}{2}u_j^0(e_j^0 + z + b_j^0) - \frac{1}{2}u_j^0(e_j^0 - z + b_j^0)\right) \tag{3.43}$$

$$\begin{aligned} q = {} & \pi(\epsilon_j^0)\frac{\partial u_j^0(e_j^0 + b_j^0)}{\partial c_j^0} \\ & + \frac{1}{2}(1-\pi(\epsilon_j^0))\left(\frac{\partial u_j^0(e_j^0 + z + b_j^0)}{\partial c_j^0} + \frac{\partial u_j^0(e_j^0 - z + b_j^0)}{\partial c_j^0}\right) \end{aligned} \tag{3.44}$$

The above FOC together with the market clearing conditions

$$\sum_{i=0}^{I} c_0^i = \sum_{i=0}^{I} e_0^i - \bar{\epsilon}^0 \tag{3.45}$$

$$\sum_{i=0}^{I}\sum_{s=1}^{3} \pi_s(\bar{\epsilon}^0)c_s^i = \sum_{i=0}^{I} \bar{e}^i \tag{3.46}$$

$$\sum_{i=0}^{I} b^i = 0 \tag{3.47}$$

[8] As with the case of complete markets, I can make similar interiority assumptions so that we do not have to look for boundary solutions.

give us the equilibrium prices and quantities and level of effort $(b^{*i}, c_0^{*i}, c_1^{*i}, c_2^{*i}, \bar{\epsilon}^*, q^*)$.[9]

Now consider a policy intervention which perturbs the holdings of the riskless bond of all agents of type 0. Again, this policy could be thought as imposing binding participation constraints on agents of type 0. The welfare effects of policy db^0 around the competitive equilibrium point are

$$
\begin{aligned}
dSW =& - d\bar{\epsilon}^0 + \pi'(\bar{\epsilon}^0)\left(\sum_{i=0}^{I}\left(u^i(c_1^i)+u^0(c_1^0)\right) - \frac{1}{2}\sum_{i=0}^{I}\left(u^i(c_2^i)-u^0(c_2^0)\right) - \frac{1}{2}\sum_{i=0}^{I}\left(u^i(c_3^i)-u^0(c_3^0)\right)\right) d\bar{\epsilon}^0 \\
&+ \pi(\bar{\epsilon}^0)\left(\sum_{i=1}^{I}\left(\frac{\partial u^i(c_1^i)}{\partial c_1^i}dc_1^i\right) + \frac{\partial u^0(c_1^0)}{\partial c_1^0}dc_1^0\right) \\
&+ \frac{1}{2}(1-\pi(\bar{\epsilon}^0))\left(\sum_{i=1}^{I}\left(\frac{\partial u^i(c_2^i)}{\partial c_2^i}dc_2^i\right) + \frac{\partial u^0(c_2^0)}{\partial c_2^0} + \sum_{i=1}^{I}\left(\frac{\partial u^i(c_3^i)}{\partial c_3^i}dc_3^i\right) + \frac{\partial u^0(c_3^0)}{\partial c_3^0}dc_3^0\right)
\end{aligned} \tag{3.48}
$$

which as before, taking into account the FOC of the agents at the equilibrium point and the market clearing conditions can be written as

$$
dSW = \pi'(\bar{\epsilon}^0)\left(\sum_{i=1}^{I} u^i(\bar{e}^i+b^i) - \frac{1}{2}\sum_{i=1}^{I} u^i(\bar{e}^i+z+b^i) - \frac{1}{2}\sum_{i=1}^{I} u^i(\bar{e}^i-z+b^i)\right) d\bar{\epsilon}^0 \tag{3.49}
$$

where

$$
d\bar{\epsilon}^0 = -\frac{\pi'(\bar{\epsilon}^0)\left(\frac{\partial u^0(\bar{e}^0+b^0)}{\partial c_1^0}dc_1^0 - \frac{1}{2}\frac{\partial u^0(\bar{e}^0+z+b^0)}{\partial c_2^0}dc_2^0 - \frac{1}{2}\frac{\partial u^0(\bar{e}^0-z+b^0)}{\partial c_3^0}dc_3^0\right)}{\pi''(\bar{\epsilon}^0)\left(u^0(\bar{e}^0+b^0) - \frac{1}{2}u^0(\bar{e}^0+z+b^0) - \frac{1}{2}u^0(\bar{e}^0-z+b^0)\right)} \tag{3.50}
$$

By strong concavity, $\sum_{i=1}^{I} u^i(\bar{e}^i+b^i) - \frac{1}{2}\sum_{i=1}^{I} u^i(\bar{e}^i+z+b^i) - \frac{1}{2}\sum_{i=1}^{I} u^i(\bar{e}^i-z+b^i) > 0$ which implies that the competitive level of effort is inefficiently low: an increase in the

[9] Equation (3.46) gives the market clearing condition of the economy in $t = 1$. The LHS is the aggregate consumption across all types of individuals: π_s can be thought as the fraction of individuals of type i that are in personal state s so that $\sum_{s=1}^{3}\pi_s(\bar{\epsilon}^0)c_s^i$ is aggregate consumption for individuals of type i.

level of effort would be welfare improving for the whole society. Then, equation (3.50) implies that the direction of the perturbation of the bond holdings of agent 0 that implements a higher level of effort depends on the sign of $(\frac{\partial u^0(\bar{e}^0+b^0)}{\partial b^0} - \frac{1}{2}\frac{\partial u^0(\bar{e}^0+z+b^0)}{\partial b^0} - \frac{1}{2}\frac{\partial u^0(\bar{e}^0-z+b^0)}{\partial b^0})$. For a concave marginal utility function $db^0 > 0:$ saving more in equilibrium is welfare improving for the whole society. Instead, for a convex marginal utility function $db^0 < 0:$ agent 0 should save less in equilibrium. In both cases, the restriction of agent 0 to save optimally induces him to exert a higher level of effort since by doing so he makes the future look less "volatile": the probability of no shock increases while the probability of shocks, positive or negative decreases which is desirable for a risk-averse agent.

3.4 Concluding Remarks and Applications

In this paper I have argued that, in economies with uncertainty and states of the world which depend on the level of effort exerted by one agent, the equilibrium is Pareto inefficient: agent 0 imposes an externality on society as he takes into account only the effect on his own welfare in his optimal choice of effort. In equilibrium, the level of effort can be inefficiently low or high depending on the aggregate welfare of all agents excluding agent 0 in each state. This result extends to economies with aggregate uncertainty and complete markets as well as to economies with uninsurable idiosyncratic risk. Therefore, although agents are price takers and they could potentially have full insurance in the case of complete markets, a market failure occurs. A more interesting question then is, whether the existing assets can be used to induce a Pareto improvement.

I show that generically in the space of endowments, there is a Pareto-improving policy in the form of a reallocation of existing assets in both the cases of complete and incomplete markets. This has the important implication that complete markets need not be optimal in an economy with probability externalities: restricting the insurance opportunities of the agents is welfare improving for the society as a whole. However, my argument does not offer any insight on what information is necessary for the determination of a Pareto improving financial intervention; it only says that one such intervention typically exists.

The above analysis provides useful insight on how to deal with big oil companies and the catastrophic risk of an oil spillage. The recent BP Deepwater Horizon oil spill has revealed glaring safety failures in both the private and the public sector. In this case, BP's unusually deep pockets made full compensation feasible. However, the question is how to create incentives for the parties involved in activities with potential environmental consequences, to take actions to prevent such events. Since big oil companies do not have full insurance against their potential pollution liability but are instead “self- insured ” by being sufficiently diversified in their portfolio investments, the above analysis suggests that a less diversified portfolio could make the society as a whole better off. The lack of a clear regulatory framework becomes even more worrying if one considers smaller firms and the possibility of bankruptcy. In that case, a small firm would be unable to cover the damages associated with an oil spillage and the damages would go unabated or most probably it would fall to the hands of the state and the general public to cover the losses and compensate the victims through public funds and taxation.

More generally, this model provides a useful framework for the discussion of how to give incentives to firms, whose actions could trigger a potential disaster, to take the necessary actions when they have limited liability. With limited liability, firms do not fully internalize the cost of their actions as they are not going to bear all the cost in the case of a disaster. One such case are nuclear plants as discussed by Eberl and Jus [34]. The basic mechanism is the fact that a nuclear plant company (NPC) cannot lose more than the legally defined liability capital or, in the worst case, its equity capital, even if the damage of a nuclear accident is much higher. This reduces the incentive to invest in costly nuclear safety and leads the NPC to a socially excessive risk level by taking into account that a share of the loss would not need to be borne by it, but could be shifted to a third party.

Finally, this model can be extended to the case of environmental externalities and natural disasters. In this case, a firm’s actions contribute to the deterioration of the environmental quality for example through CO_2 emissions and the subsequent increase in temperature. Temperature change is considered to be the leading factor for the increase in the number of extreme events such as droughts and floods. As a consequence, a firm is affecting the probability of extreme events or in other words, the state of the world. However, in its choice of effort, for example investing in cleaner technologies, the firm takes into account the effect only on its own welfare and fails to internalize the social cost. In this case, my analysis suggests that restricting the insurance opportunities of the firm makes it more vulnerable to the risk and creates incentives for higher level of effort.

Chapter 4

Environmental Externalities and Fiscal Policy

(with A. Carvajal and J. Dávila)

The role of public policy in environmental problems is a hotly debated issue among economists and non economists alike. The only consensus seems to be that if there is to be public intervention it has to come as an incentive-based instrument rather than command and control. The early "Pigouvian" literature on externalities and corrective taxes has argued that it would be sufficient to levy a tax on an externality generating activity equal to its social marginal damage. However, Pigou taxes would be sufficient to achieve Pareto optimality only in a first best environment. In a second best setting, where there are other distortionary taxes in the system, this prescription must be modified. The problem was originally studied by Sandmo [88] and more recently by Bovenberg and Van der Ploeg [12]. What they have shown is what Sandmo called the "additivity property" where the presence of externality only changes the tax formula for the externality generating good but leaves the optimal income and commodity taxes unaffected. Bovenberg and Van der Ploeg [13] and Bovenberg and de Mooiz [12] conclude that "in the presence of pre-existing distortionary taxes, the optimal pollution tax typically lies below the Pigouvian tax...". In other words, while the first best rule calls for a tax that is only corrective, the second best tax embodies both corrective and optimal tax objectives, a tax structure which may be described as "Ramsey plus Pigou". Following the above literature, Cremer, Gahvari and Ladoux [24] and Cremer, Gahvari and Ladoux [25] allow for more than one polluting goods and for non linear taxation casting light on the importance and the relevance of the separability restrictions on preferences required for the above result and concluding that the optimal environmental levy amy exceed, fall short of, or be equal to the Pigouvian tax depending on the assumptions made.

What this literature is lacking however, is a more general treatment of the economy, in terms of the preferences, the type of externalities and the available correcting mechanisms. In economies with heterogeneous preferences, tax intervention is often said to be counterproductive because competitive equilibria cannot be improved by anonymous taxes. In this direction, Geanakoplos and Polemarchakis [40] are the first to offer a generic analysis of competitive equilibria and optimality in economies with externalities. They show that, generically in the space of exchange economies with linear, separable consumption externalities and heterogeneous preferences, competitive equilibria exist and are constrained inefficient: there exists a policy intervention, in the form of anonymous taxes on the purchases of commodities, and anonymous, uniform transfers of revenue, which are balanced from a fiscal point of view that induce welfare improvements for all agents in the economy. While this is the first paper that explores the existence of anonymous Pareto-improving taxes in an economy with externalities, the analogous question with different market failures such market incompleteness and moral hazard, has been analyzed repeatedly. With uncertainty and an incomplete asset market, Geanakoplos and Polemarchakis [40] proved the constrained suboptimality of competitive equilibrium allocations: generically, there is a reallocation of assets that leads to a Pareto superior allocation of goods after prices in commodity spot markets adjust and markets clear. Citanna et al. [22] showed that taxation, which is anonymous, could induce a Pareto improving reallocation of assets. Carvajal and Polemarchakis [16] extend these results to economies with no aggregate uncertainty and uninsurable idiosyncratic risk where the reallocation of assets works through its effects on future relative prices. They also provide examples of how their argument could be applied in an OLG economy.

The purpose of this paper is to provide a model to study the existence of anonymous Pareto-improving taxes and subsidies in the generic space of exchange economies with heterogeneous agents. We plan to apply the results of Geanakoplos and Polemarchakis [40] to the more stylized framework used in environmental economics, in order to bring to the attention of environmental economists the more recent developments on market failure theory. This will require us to extend the original results to production economies, while adding much generality to the framework used, for instance, by Sandmo. Importantly, we will also study the case of externalities in an OLG with heterogeneous agents, setting, extending and applying the results of Carvajal and Polemarchakis [16] where the market failure is instead through a missing market for securities.

In the next sections, we present a benchmark model of an infinite-horizon production economy where production activities generate pollution. We solve for the competitive equilibrium and we show that as in all economies with externalities, the equilibrium is Pareto inefficient. We then focus on the design of the optimal fiscal policy for the implementation of the first-best. We find that a mix of tax on capital and labor, combined with a subsidy on afforestation and lump-sum transfers implement the first-best allocation. Next, we will build upon this model to allow for heterogeneous agents and an OLG setup. In such a framework, the lack of information makes the decentralization of the first best infeasible. Instead, one should ask if there are any Pareto improving policies at all. The focus is then on proving the constrained inefficiency of the competitive equilibrium. The extension of the model to allow for heterogeneous preferences and an OLG setup is part of our ongoing research.

4.1 The Economy

Consider an infinite-horizon economy. A representative household derives utility from consumption, c_t, and from leisure. Denoting by l_t the labor supplied by the household, we model its preferences as

$$u(c_t, \gamma_t) - v(l_t, \gamma_t),$$

where γ_t represents the state of the world's environment. We interpret this state as the amount of greenhouse gases accumulated in the atmosphere, so that

$$u_\gamma < 0,\ v_\gamma > 0,\ u_{c\gamma} < 0, \text{ and } v_{l\gamma} > 0.$$

Output, y_t, is produced using capital, k_t, and labor, with a constant-returns-to-scale technology

$$y_t = A(\gamma_t)F(K_t, l_t),$$

where the total factor productivity depends negatively on the stock of greenhouse gases, $A' < 0$. The intuition behind this assumption is that a deteriorating natural environment negatively affects productivity of the direct factors of production, e.g. by increasing natural depreciation, reducing the quality of natural inputs or by deteriorating the health of workers.

Output can be used for consumption, for capital accumulation, or for afforestation (and reforestation) activities, a_t. Assuming full capital depreciation, the aggregate resources constraint is that

$$c_t + k_{t+1} + a_t \leq y_t.$$

Production activities generate pollution, p_t. In order to allow for the possibility that capital and labor produce different greenhouse gas emissions, we model

$$p_t = P(k_t, l_t),$$

where

$$P_k > 0, \text{ and } P_l \geq 0.$$

Also, we impose the condition that

$$P_l - \frac{F_l}{F_k} P_k \leq 0,$$

which guarantees that the substitution of capital for labor along an output isoquant does not increment greenhouse gas emissions. Therefore, labor is considered a "cleaner" technology than capital. [1]

The state of the environment evolves according to the difference equation

$$\gamma_{t+1} = \gamma_t + p_t - S(\gamma_t, a_t),$$

where S represents the sequestration of greenhouse gases. We are assuming that, $S_a > 0$ which means that increasing afforestation activities increase sequestration of greenhouse gas emissions, $S_\gamma < 0$, meaning that the worse the state of the environment is, the less efficient sequestration is and $S(\gamma_t, a_t)$ concave implying decreasing efficiency of sequestration.

4.2 Competitive Equilibrium

Denote the prices of capital and labor as r_t and w_t, respectively.

[1] A direct implication of this assumption is that the use of the "cleaner" factor, labor, instead of "dirty" capital will have as a result a lower level of overall taxation and therefore there are clear incentives to use the c This is shown in what follows for a situation in which we are in a steady state. Denote with Δ the change in total taxation then

$$\Delta = \tau^w dl + \tau^r dk$$

and we require that $\Delta \leq 0$. Notice that in order to have no change at the level of production we need that $dk = -dl \frac{F_l}{F_k}$ which we substitute in the above expression and we get

$$\left(\tau^w - \tau^r \left(\frac{F_l}{F_k}\right)\right) dl = \Delta$$

Therefore for $\Delta \leq 0$ we need that

$$\tau^w \leq \tau^r \left(\frac{F_l}{F_k}\right)$$

or

$$\frac{P_l}{S_a} \leq \frac{P_k}{S_a} \left(\frac{F_l}{F_k}\right)$$

which is the initial assumption that we have made.

4.2.1 The competitive household

The household takes as given the sequence $\{r_t, w_t\}$ of prices, as well the sequence $\{\gamma_t\}$ that represents the evolution of the stock of greenhouse gases. Given these sequences, it chooses the plan $\{l_t, c_t, a_t, k_{t+1}\}$ so as to maximize its discounted utility,

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, \gamma_t) - v(l_t, \gamma_t)],$$

subject to the budget constraint that

$$c_t + k_{t+1} + a_t \leq w_t l_t + r_t k_t.$$

It is immediate from the formulation of the problem that the household *optimally* chooses $a_t = 0$ at all periods. The first-order conditions of its problem are hence that, at all periods,

$$\beta^t u_c(c_t, \gamma_t) = \lambda_t,$$
$$\beta^t v_l(l_t, \gamma_t) = \lambda_t w_t,$$
$$\lambda_t = \lambda_{t+1} r_{t+1},$$

for a sequence $\{\lambda_t\}$ of positive numbers, while

$$c_t + k_{t+1} = w_t l_t + r_t k_t,$$

with k_0 given.

4.2.2 The competitive firm

The firm also takes as given the sequences $\{r_t, w_t\}$ and $\{\gamma_t\}$, and chooses its demands for capital and labor, at each period, so as to maximize

$$y_t - w_t l_t - r_t k_t,$$

subject to the technological constraint that

$$y_t = A(\gamma_t) F(k_t, l_t),$$

Substituting the constraint into the objective function, we can rewrite the problem as choosing a sequence $\{l_t, k_t\}$ so as to maximize

$$A(\gamma_t)F(k_t, l_t) - w_t l_t - r_t k_t$$

at each period. The first-order conditions of this problem are that

$$A(\gamma_t)F_l(k_t, l_t) = w_t,$$
$$A(\gamma_t)F_k(k_t, l_t) = r_t.$$

4.2.3 The equilibrium

The following conditions characterize the evolution of sequence $\{c_t, l_t, k_{t+1}, \gamma_{t+1}\}$ at a competitive equilibrium:

$$\frac{v_l(l_t, \gamma_t)}{u_c(c_t, \gamma_t)} = A(\gamma_t)F_l(k_t, l_t), \tag{CE1}$$

$$\frac{u_c(c_t, \gamma_t)}{\beta u_c(c_{t+1}, \gamma_{t+1})} = A(\gamma_{t+1})F_k(k_{t+1}, l_{t+1}), \tag{CE2}$$

$$c_t + k_{t+1} = A(\gamma_t)F(k_t, l_t), \tag{CE3}$$

$$\gamma_{t+1} = \gamma_t + P(k_t, l_t) - S(\gamma_t, 0), \tag{CE4}$$

given initial conditions (k_0, γ_0).

4.3 First Best

A central planner would choose the sequence $\{\bar{c}_t, \bar{k}_{t+1}, \bar{l}_t, \bar{a}_t, \gamma_t\}$ so as to maximize the discounted household utility

$$\sum_{t=0}^{\infty} \beta^t [u(\bar{c}_t, \gamma_t) - v(\bar{l}_t, \gamma_t)]$$

subject to the aggregate resources constraint that

$$\bar{c}_t + \bar{k}_{t+1} + \bar{a}_t \leq A(\gamma_t)F(\bar{k}_t, \bar{l}_t)$$

and to the evolution of the state of the environment,

$$\gamma_{t+1} = \gamma_t + P(\bar{k}_t, \bar{l}_t) - S(\gamma_t, \bar{a}_t),$$

given initial conditions $(\bar{k}_0, \gamma_0) = (k_0, \gamma_0)$.

The first-order conditions of this problem are that, at all periods,

$$\begin{pmatrix} \beta^t u_c(c_t,\gamma_t) \\ 0 \\ -\beta^t v_l(l_t,\gamma_t) \\ 0 \\ \beta^t[u_\gamma(c_t,\gamma_t) - v_\gamma(l_t,\gamma_t)] \end{pmatrix} = \delta_t \begin{pmatrix} 1 \\ 1 \\ -A(\gamma_t)F_l(k_t,l_t) \\ 1 \\ -A'(\gamma_t)F(k_t,l_t) \end{pmatrix} + \delta_{t+1} \begin{pmatrix} 0 \\ -A(\gamma_{t+1})F_k(k_{t+1},l_{t+1}) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$+\mu_{t-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \mu_t \begin{pmatrix} 0 \\ 0 \\ P_l(k_t,l_t) \\ -S_a(\gamma_t,a_t) \\ 1 - S_\gamma(\gamma_t,a_t) \end{pmatrix} + \mu_{t+1} \begin{pmatrix} 0 \\ P_k(k_{t+1},l_{t+1}) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

that is to say

$$\begin{aligned}
&\beta^t u_c(\bar{c}_t,\gamma_t) = \delta_t \\
&\beta^t v_l(\bar{l}_t,\gamma_t) = \delta_t A(\gamma_t)F_l(\bar{k}_t,\bar{l}_t) - \mu_t P_l(\bar{k}_t,\bar{l}_t) \\
&\delta_t = \delta_{t+1}A(\gamma_{t+1})F_k(\bar{k}_{t+1},\bar{l}_{t+1}) - \mu_{t+1}P_k(\bar{k}_{t+1},\bar{l}_{t+1}) \\
&\delta_t = \mu_t S_a(\gamma_t,\bar{a}_t) \\
&\beta^t[u_\gamma(c_t,\gamma_t) - v_\gamma(l_t,\gamma_t)] = -\delta_t A'(\gamma_t)F(k_t,l_t) - \mu_{t-1} + \mu_t(1 - S_\gamma(\gamma_t,a_t))
\end{aligned}$$

for a sequence $\{\delta_t,\mu_t\}$ of numbers, where δ_t is positive at all periods, as well as the equalities

$$\begin{aligned}
\bar{c}_t + \bar{k}_{t+1} &= A(\bar{\gamma}_t)F(\bar{k}_t,\bar{l}_t), \\
\bar{\gamma}_{t+1} &= \bar{\gamma}_t + P(\bar{k}_t,\bar{l}_t) - S(\bar{\gamma}_t,\bar{a}_t).
\end{aligned}$$

Solving for the multipliers from the first-order system, we have that the following conditions characterize the evolution of sequence $\{\bar{c}_t, \bar{l}_t, \bar{k}_{t+1}, \bar{a}_t, \bar{\gamma}_{t+1}\}$ at the first best:

$$\frac{v_l(\bar{l}_t, \bar{\gamma}_t)}{u_c(\bar{c}_t, \bar{\gamma}_t)} = A(\bar{\gamma}_t)F_l(\bar{k}_t, \bar{l}_t) - \frac{P_l(\bar{k}_t, \bar{l}_t)}{S_a(\bar{\gamma}_t, \bar{a}_t)} \tag{FB1}$$

$$\frac{u_c(\bar{c}_t, \bar{\gamma}_t)}{\beta u_c(\bar{c}_{t+1}, \bar{\gamma}_{t+1})} = A(\bar{\gamma}_{t+1})F_k(\bar{k}_{t+1}, \bar{l}_{t+1}) - \frac{P_k(\bar{k}_{t+1}, \bar{l}_{t+1})}{S_a(\bar{\gamma}_{t+1}, \bar{a}_{t+1})} \tag{FB2}$$

$$\begin{aligned}\frac{v_\gamma(c_t, \gamma_t) - u_\gamma(c_t, \gamma_t)}{u_c(c_t, \gamma_t)} &= A'(\gamma_t)F(k_t, l_t)\\ &+ \frac{v_l(l_{t-1}, \gamma_{t-1}) - u_c(l_{t-1}, \gamma_{t-1})A(\gamma_{t-1})F_l(k_{t-1}, l_{t-1})}{\beta u_c(c_t, \gamma_t)P_l(k_{t-1}, l_{t-1})}\\ &- \frac{1 - S_\gamma(\gamma_t, a_t)}{S_a(\gamma_t, a_t)}\end{aligned} \tag{FB2.5}$$

$$\bar{c}_t + \bar{k}_{t+1} + \bar{a}_t = A(\bar{\gamma}_t)F(\bar{k}_t, \bar{l}_t) \tag{FB3}$$

$$\bar{\gamma}_{t+1} = \bar{\gamma}_t + P(\bar{k}_t, \bar{l}_t) - S(\bar{\gamma}_t, \bar{a}_t), \tag{FB4}$$

given the initial conditions $(\bar{k}_0, \bar{\gamma}_0) = (k_0, \gamma_0)$.

4.4 Implementation of the First Best

It is immediate from the comparison of (CE1)–(CE2) and (FB1)–(FB2) that in the presence of pollution the competitive equilibrium allocation is inefficient —not to mention that (FB2.5) needs not hold at a competitive equilibrium. This is because the competitive decisions on capital and labor usage, and on expenditure in afforestation ignore their effects on the state of the environment.

4.4.1 Competitive equilibrium under intervention

Suppose that a fiscal policy levies taxes at rates τ_t^w and τ_t^r on the labor and capital revenues of the household, and distributes lump-sum transfers T_t to it. Suppose also that an environmental policy specifies a schedule $\sigma_t(a)$ of subsidies that the household will receive if it spends the amount a of its revenue in afforestation.

Then, the competitive household is going to choose plan $\{\tilde{l}_t, \tilde{c}_t, \tilde{a}_t, \tilde{k}_{t+1}\}$ so as to maximize

$$\sum_{t=0}^{\infty} \beta^t [u(\tilde{c}_t, \gamma_t) - v(\tilde{l}_t, \gamma_t)],$$

subject to the budget constraint that

$$\tilde{c}_t + \tilde{k}_{t+1} + \tilde{a}_t \leq (w_t - \tau_t^w)\tilde{l}_t + (r_t - \tau_t^k)\tilde{k}_t + T_t + \sigma_t(\tilde{a}_t).$$

The first order conditions of this problem require that

$$\begin{aligned}
&\beta^t u_c(\tilde{c}_t, \gamma_t) = \tilde{\lambda}_t, \\
&\beta^t v_l(\tilde{l}_t, \gamma_t) = \tilde{\lambda}_t (w_t - \tau_t^w), \\
&\tilde{\lambda}_t = \tilde{\lambda}_{t+1}(r_{t+1} - \tau_{t+1}^r), \\
&\sigma_t'(\tilde{a}_t) = 1.
\end{aligned}$$

for a sequence $\{\tilde{\lambda}_t\}$ of positive numbers, while

$$\tilde{c}_t + \tilde{k}_{t+1} + \tilde{a}_t = (w_t - \tau_t^w)\tilde{l}_t + (r_t - \tau_t^k)\tilde{k}_t + T_t + \sigma_t(\tilde{a}_t).$$

The first-order conditions of the competitive firm remain the same, whereas the evolution of the stock of greenhouse gases is now given by

$$\gamma_{t+1} = \gamma_t + P(\tilde{k}_t, \tilde{l}_t) - S(\gamma_t, \tilde{a}_t),$$

and the budget balancedness of fiscal policy requires that

$$\tau_t^w \tilde{l}_t + \tau_t^k \tilde{k}_t - \sigma_t(\tilde{a}_t) = T_t.$$

As before, we can characterize the sequence $\{\tilde{c}_t, \tilde{l}_t, \tilde{k}_{t+1}, \tilde{\gamma}_{t+1}\}$ at a competitive equilibrium under intervention:

$$\begin{aligned}
&\frac{v_l(\tilde{l}_t, \tilde{\gamma}_t)}{u_c(\tilde{c}_t, \tilde{\gamma}_t)} = A(\tilde{\gamma}_t) F_l(\tilde{k}_t, \tilde{l}_t) - \tau_t^w, && \text{(CEI1)} \\
&\frac{u_c(\tilde{c}_t, \tilde{\gamma}_t)}{\beta u_c(\tilde{c}_{t+1}, \tilde{\gamma}_{t+1})} = A(\tilde{\gamma}_{t+1}) F_k(\tilde{k}_{t+1}, \tilde{l}_{t+1}) - \tau_t^r, && \text{(CEI2)} \\
&\sigma_t'(\tilde{a}_t) = 1, && \text{(CEI3)} \\
&\tilde{c}_t + \tilde{k}_{t+1} + \tilde{a}_t = A(\tilde{\gamma}_t) F(\tilde{k}_t, \tilde{l}_t), && \text{(CEI4)} \\
&\tau_t^w \tilde{l}_t + \tau_t^k \tilde{k}_t - \sigma_t(\tilde{a}_t) = T_t, && \text{(CEI5)} \\
&\tilde{\gamma}_{t+1} = \tilde{\gamma}_t + P(\tilde{k}_t, \tilde{l}_t) - S(\tilde{\gamma}_t, \tilde{a}_t), && \text{(CEI6)}
\end{aligned}$$

given initial conditions $(\tilde{k}_0, \tilde{\gamma}_0) = (k_0, \gamma_0)$.

4.4.2 Optimal intervention

Since the first best is characterized by

$$\frac{v_l(\bar{l}_t,\bar{\gamma}_t)}{u_c(\bar{c}_t,\bar{\gamma}_t)} = A(\bar{\gamma}_t)F_l(\bar{k}_t,\bar{l}_t) - \frac{P_l(\bar{k}_t,\bar{l}_t)}{S_a(\bar{\gamma}_t,\bar{a}_t)} \tag{FB1}$$

$$\frac{u_c(\bar{c}_t,\bar{\gamma}_t)}{\beta u_c(\bar{c}_{t+1},\bar{\gamma}_{t+1})} = A(\bar{\gamma}_{t+1})F_k(\bar{k}_{t+1},\bar{l}_{t+1}) - \frac{P_k(\bar{k}_{t+1},\bar{l}_{t+1})}{S_a(\bar{\gamma}_{t+1},\bar{a}_{t+1})} \tag{FB2}$$

$$\begin{aligned}\frac{v_\gamma(c_t,\gamma_t) - u_\gamma(c_t,\gamma_t)}{u_c(c_t,\gamma_t)} &= A'(\gamma_t)F(k_t,l_t)\\ &+ \frac{v_l(l_{t-1},\gamma_{t-1}) - u_c(l_{t-1},\gamma_{t-1})A(\gamma_{t-1})F_l(k_{t-1},l_{t-1})}{\beta u_c(c_t,\gamma_t)P_l(k_{t-1},l_{t-1})}\\ &- \frac{1-S_\gamma(\gamma_t,a_t)}{S_a(\gamma_t,a_t)}\end{aligned} \tag{FB2.5}$$

$$\bar{c}_t + \bar{k}_{t+1} + \bar{a}_t = A(\bar{\gamma}_t)F(\bar{k}_t,\bar{l}_t) \tag{FB3}$$

$$\bar{\gamma}_{t+1} = \bar{\gamma}_t + P(\bar{k}_t,\bar{l}_t) - S(\bar{\gamma}_t,\bar{a}_t), \tag{FB4}$$

given the initial conditions $(\bar{k}_0,\bar{\gamma}_0) = (k_0,\gamma_0)$, where (FB2.5) can be written as

$$\Phi(\gamma_t,k_t,l_t,\gamma_{t-1},k_{t-1},l_{t-1})S_a(\gamma_t,a_t) + S_\gamma(\gamma_t,a_t) = 1$$

with

$$\begin{aligned}\Phi(\gamma_t,k_t,l_t,\gamma_{t-1},k_{t-1},l_{t-1},c_t) &= A'(\gamma_t)F(k_t,l_t)\\ &+ \frac{v_l(l_{t-1},\gamma_{t-1}) - u_c(l_{t-1},\gamma_{t-1})A(\gamma_{t-1})F_l(k_{t-1},l_{t-1})}{\beta u_c(c_t,\gamma_t)P_l(k_{t-1},l_{t-1})}\\ &- \frac{v_\gamma(c_t,\gamma_t) - u_\gamma(c_t,\gamma_t)}{u_c(c_t,\gamma_t)}\end{aligned} \tag{4.1}$$

Consider the following intervention. Let the fiscal policy be [2]

$$\begin{aligned}\tau_t^w &= \frac{P_l(\bar{k}_t,\bar{l}_t)}{S_a(\bar{\gamma}_t,\bar{a}_t)}\\ \tau_t^r &= \frac{P_k(\bar{k}_{t+1},\bar{l}_{t+1})}{S_a(\bar{\gamma}_{t+1},\bar{a}_{t+1})}\\ \sigma_t(a_t) &= \Phi(\gamma_t,k_t,l_t,\gamma_{t-1},k_{t-1},l_{t-1},c_t)S(\gamma_t,a_t) + \int^{a_t} S_\gamma(\gamma_t,a)da\\ T_t &= \frac{P_l(\bar{k}_t,\bar{l}_t)}{S_a(\bar{\gamma}_t,\bar{a}_t)}\bar{l}_t + \frac{P_k(\bar{k}_{t+1},\bar{l}_{t+1})}{S_a(\bar{\gamma}_{t+1},\bar{a}_{t+1})}\bar{k}_t - \sigma_t(\bar{a}_t)\end{aligned}$$

[2] Notice that σ_t depends explicitly only on a_t and, specifically, not on l_t, c_t: this is meant to convey that agents are not aware of these dependencies when they maximize

It is immediate that (CEI3) implies (FB2.5), whereas the definition of T_t implies condition (CEI5), provided that $(\tilde{k}_t, \tilde{l}_t) = (\bar{k}_t, \bar{l}_t)$. Now, equations (CEI1), (CEI2), (CEI4) and (CEI6) are equivalent to (FB1), (FB2), (FB3) and (FB4), respectively, which suffices to imply that the policy intervention implements the first best allocation.

4.5 Conclusions and Further Research

We have presented a benchmark model of an infinite-horizon production economy where production activities generate pollution. We solved for the competitive equilibrium and showed that as in all economies with externalities, the equilibrium is Pareto inefficient. We then focused on the design of the optimal fiscal policy for the implementation of the first-best. We found that a mix of tax on capital and labor, combined with a subsidy on afforestation and lump-sum transfers implement the first-best allocation.

The next step is to extend this model to allow for heterogeneous agents and an OLG setup. In such a framework, the lack of information makes the decentralization of the first best infeasible, an argument which has been often used by supporters of no intervention. However, the real question one should ask is whether there are any Pareto improving policies at all. The focus is then on proving the constrained inefficiency of the competitive equilibrium which is to say that although the first best allocation cannot be achieved, a carefully designed environmental policy can still make everybody better off.

Chapter 5

Temptation Driven Preferences and Environmental Externalities

5.1 Introduction

The idea that agents might be tempted by immediate gratification has been the benchmark of the literature on self control and time inconsistent behavior. Strotz [94] was the first one to suggest a model where the agent's future behavior is inconsistent with his optimal plan giving rise to a demand for pre-commitment devices. Phelps and Pollak [79] study second best national saving when the present generation lacks the power to commit future generations' decisions while Laibson [62] adopts the same framework to model time-inconsistency within an individual in the presence of an imperfect commitment technology. In the same spirit, O'Donoghue and Rabin [75] explore the welfare and behavioral implications of present biased preferences when an agent has to engage in an activity exactly once during some length of time while O'Donoghue and Rabin [76] extent the model to accommodate for the choice of which task to do and for partial naiveté.

In these models of time-inconsistent preferences, there is a sequence of the consumer's different "selves" who value consumption streams in an unique way and play a dynamic game. In contrast and following the work on preference for flexibility of Kreps [59], Gul and Pesendorfer [46] are the first to develop a theoretical framework and axiomatization for preferences over menus when agents suffer from temptation without necessitating spitting up the agent into multiple selves. In particular, they build a 2-period decision problem where in the first period agents choose over menus of lotteries and in the second period they choose an alternative from the chosen menu. However, agents are subject to temptation: at the time of actual consumption, they suffer from urges to deviate from their "commitment" preferences which prescribe what they "should" do and instead evaluate alternatives according to their "temptation" preferences which is what they "want" to do. Importantly, even if they resist temptation, they will suffer from a self-control cost. Their work is closely related to that of Dekel, Lipman and Rustichini [28], while Dekel, Lipman and Rustichini [29] offer a natural generalization of their representation theorem to incorporate uncertainty about future temptations and multiple temptations. Stovall [93] deals with uncertainty regarding future temptations too but retains the assumption that the agent is tempted only by the most tempting alternative of a given menu. Gul and Pesendorfer [47] employ an infinite horizon model to define and characterize dynamic self-control preferences and to study the optimality of the equilibria attained while Noor [68, 69] extends their model to accommodate for temptation by future consumption.

However, there are only a few applications of the above framework. Amador, Werning and Angeletos [5] study the optimal trade-off between commitment and flexibility in a consumption-savings model where agents are subject to taste shocks while Krusell, Kuruscu and Smith [61] study optimal taxation when consumers have temptation and self-control problems and show that it is possible to improve utility even with a linear tax-transfer scheme. Miao [66] analyses an agent's decision to exercise an option under uncertainty when he is tempted by immediate gratification and suffers from self-control problems. Esteban, Miyagawa and Shum [37] characterize optimal non-linear pricing for a monopoly when self-control is costly for consumers.

This paper aims to model an economy where agents, endowed with temptation and self control preferences, take actions that create externalities for the whole society. In particular, I will consider the problem of environmental pollution as a byproduct of consumption. I analyze a two- period, two-countries model where in the first period countries, represented by their decision makers, negotiate over the upper(and potentially lower) bounds of consumption and therefore over the associated level of pollution. In the second period, the consumers choose a level of consumption within the allowed range. In this context and with agents experiencing different types of temptation or random temptation, I find that it is optimal for a decision maker to commit to a singleton set avoiding in this way temptation and the cost of self-control. Allowing for shocks in the economy creates a trade-off between commitment and flexibility to adjust to shocks. In this case, the optimal policy for a decision maker will depend on the range of parameters. I find that for a relatively small shock, full commitment is favored at the expense of no adjustment to shocks. Instead, for a relatively big shock, uncertainty becomes too important to ignore and some degree of flexibility is optimal at the expense of a self-control cost.

Up to now, the model has offered an insight into how a decision maker picks a choice set to maximize exante utility taking into account the choice behavior of an agent at the time of actual consumption. However, his choice does nothing to correct the externality created by aggregate consumption. Thus, the next natural step is to discuss the social welfare implications of this model and provide a definition for social optimality when agents are endowed with temptation preferences. The discussion would then focus on how to internalize the cost of the externality and implement the socially optimal allocation. A natural extension of this model is to allow for strategic interactions between the countries. In this case, the optimal choice of a consumption set would not only reflect the desire to reduce the self control cost associated with temptation but also strategic considerations: it will be the optimal response to the choice set of the other players. It would be interesting then to study whether, with the particular type of preferences, there are Nash Equilibria other than the standard Prisoner's Dilemma non-cooperative outcome. The internalization of the externality cost as well as the extension to a game setup are part of my future research.

The rest of the paper is organized as follows. Section 2 presents the benchmark model and background as developed by Gul and Pesendorfer [46]. In section 3, I solve for the competitive equilibrium and the Pareto optimal allocation in an economy where agents have standard preferences. In section 4, I extend my analysis in order to incorporate agents with "temptation and self-control preferences" while allowing for shocks in the economy. Section 5 is devoted to conclusions and further research.

5.2 The Model

My model is based on the framework first suggested by Gul and Pesendorfer [46], therefore we start with an analysis of their setup. The core idea of their framework is that agents suffer from temptation: at the time of consumption, they might be tempted to make an ex ante inferior choice. As a result, agents of this type might prefer a"restricted" menu in the sense of removing tempting alternatives and escaping any chance of deviation from their ex ante optimal behavior. Therefore, they make preferences over menus, rather than single alternatives, the primitive of their model. They consider a two period decision problem where in the first period agents choose over menus of lotteries while in the second, they have to pick an alternative from the chosen menu. The two building blocks of their model is what they call a "commitment" utility and a "temptation" utility. The "commitment" utility describes the normative preferences of an agent in the first period: it is the utility he attaches to alternatives when he is in a "cool" state and therefore not subject to temptation. It is also the utility he attaches to singleton sets as in that case there is no other tempting alternative. On the other hand, the "temptation" utility describes the agent's "urges" in the second period: it is the utility he attaches to alternatives when he is in a "hot" state and tempted to pick an ex ante inferior alternative. In this framework, the utility that the agent attaches to a menu A is given by

$$U(A) = \max_{x \in A}[u(x) + v(x)] - \max_{y \in A} v(y)$$

where u is the "commitment" utility and v is the " temptation" utility. Both u and v are von Neumann-Morgenstern utility functions over lotteries. The above representation describes how an agent evaluates menus: welfare is given by the maximized value of the sum of the commitment and the temptation utility minus the temptation utility evaluated at the most tempting alternative of the menu. It also suggests a choice behavior in the second period: given a menu A, the agent's actual choice maximizes $u(x)+v(x)$ while at the same time he experiences a cost of self-control given by $\max_{y \in A} v(y)$. Therefore, his second period choice behavior represents an optimal compromise between the utility that could have been achieved under commitment and the cost of self-control.

Going back to my framework, the aim of this paper is to model an economy where agents, endowed with temptation and self control preferences, take actions that create externalities for the whole society. In particular, I will consider the problem of environmental pollution as a byproduct of consumption. As a benchmark, I consider a two period economy and two countries where agents act as "decision makers" in the first period and as "consumers" in the second period. I denote countries and their representative agent with i where $i = 1, 2$. In the first period, the choice period, the representative agent of country i, acting as a "decision maker" negotiates over the lower and upper bounds of consumption $\underline{q}_i$ and $\bar{q}_i \in \mathbb{R}_+$ and implicitly over the associated level of pollution. In the second period, the consumption period, the agents acting as "consumers" choose an actual level of consumption q_i within the bounds the decision maker has committed to. Therefore, the preferences of the decision maker are defined over consumption intervals or choice sets, taking into account the implied second period behavior of the agents of country i and taking as given the consumption of all the other countries q_{-i}. The decision maker of country i evaluates each consumption interval with the following utility index

$$U_i(\bar{q}_i, q_{-i}) = \sum_{n=1}^{N} p_{in}\Big[\max_{\underline{q}_i \leq q_i \leq \bar{q}_i}[u_i(q_i, q_{-i}) + v_{in}(q_i, q_{-i})] - \max_{\underline{q}_i \leq q_i \leq \bar{q}_i} v_{in}(q_i, q_{-i})\Big] \tag{5.1}$$

where both u_i *and* v_{in} are strictly increasing and strictly concave and therefore $u_i + \sum_{n=1}^{N} v_{in}(q_i, q_j)$ is also strictly increasing and strictly concave, q_i is actual consumption of a representative agent of country i and q_{-i} is consumption of all other countries. This representation of preferences, first suggested by Dekel, Lipman and Rustichini [29] and Stovall [93] is an extension of Gul and Pesendorfer [46] which allows for uncertainty regarding the type of temptation that can affect an agent at the time of consumption. As before, u_i represents the "commitment" utility and v_{in} represents the "temptation" utility associated with a temptation denoted by n, with $n = 1, 2, \ldots N$. Therefore, in the second period and after the decision maker has committed to a lower and upper bound of consumption, an agent subject to random temptation evaluates alternatives according to $u_i(q_i, q_{-i}) + v_{in}(q_i, q_{-i})$ with probability p_{in} while he suffers the cost of self-control which is given $\max_{\underline{q}_i \leq q_i \leq \bar{q}_i} v_{in}(q_i, q_{-i})$. As implied by the representation of preferences in (5.1), when choosing the bounds for consumption, a decision maker takes into account this optimizing behavior of the consumers and the associated cost of self-control.

As a benchmark for my analysis, I start with an economy with agents endowed with standard preferences and I solve for the competitive equilibrium and the Pareto optimal allocation. Then, I move on to models where agents have "non-standard" preferences, what we have called "temptation and self-control preferences".

5.3 A representative agent with standard preferences

As a benchmark for our analysis, consider an economy with 2 countries and only one consumption good: denote each country with i, where $i = 1, 2$. Agents have standard preferences: they are not subject to temptation in the second period, the period of actual consumption. Therefore, they have the same preferences in the first and the second period, which are given by

$$u_i(q_1, q_{-i}) = w_i(q_i) - \phi_i(q_i, q_{-i}) \tag{5.2}$$

where $w_i(q_i)$ is strictly increasing and strictly concave, $\phi_i(q_i, q_{-i})$ is strictly increasing and convex, q_i is consumption of the representative agent of country i and q_{-i} is consumption of the representative agent of the other country, with q_i and $q_{-i} \in \mathbb{R}_+$. I will further assume a simple linear form for $\phi_i(q_i, q_{-i})$

$$\phi_i(q_i, q_{-i}) = \epsilon_i(q_i + q_{-i}) \tag{5.3}$$

with $\epsilon_i > 0$.[1] I will call u_i the "commitment" utility function of a representative agent of country i: it describes his preferences when he does not suffer from any temptation or when he is in a "cool" state. The above representation of preferences implies that there are costs associated with consumption: agents derive disutility from pollution, a direct byproduct of consumption as described by ϕ_i. These costs could be associated with lower environmental standards and associated health risks, reduced productivity or even with a reduced value of the environment as a pure amenity. For any of these reasons, agents have environmental concerns which are represented by the parameter ϵ_i and face a trade-off when they decide for the optimal level on consumption. However, as I will show in the next section, they only take into account the effects of a lower environmental quality on themselves, creating therefore a source of inefficiency.

[1] In this version of my model, I want to avoid any strategic interaction between agents, thus I employ a linear functional form or a functional form without any interaction terms, for the cost function ϕ.The case of strategic interaction in a game-theory setup becomes interesting in the presence of uncertainty and it is the subject of my future research.

5.3.0.1 Competitive Equilibrium and Pareto Optimality

A representative agent of country i, taking q_{-i} as given, solves the following maximization problem

$$\max_{q_i}[w_i(q_i) - \epsilon_i(q_i + q_{-i})] \tag{5.4}$$

with

$$\frac{\partial w_i(q_i)}{\partial q_i} = \epsilon_i \tag{5.5}$$

Then, a competitive equilibrium of this economy, is a pair (q_1^*, q_2^*) where q_1^* and q_2^* are the solutions of the FOC for countries 1 and 2. As a result of the linear functional form the externality generating function, there is no strategic interaction at this stage.

One way of solving for the Pareto optimum is to guarantee a minimum amount of utility for agent 2 and then maximise agent 1's utility subject to this constraint. In other words, an allocation $(\hat{q}_1, \hat{q}_2)$ is Pareto optimal if it solves

$$\max_{q_1,q_2}[w_1(q_1) - \epsilon_1(q_1 + q_2)] \tag{5.6}$$

$$\text{st. } w_2(q_2) - \epsilon_2(q_1 + q_2) \geq w_2(\hat{q}_2) - \epsilon_2(\hat{q}_2 + \hat{q}_1)$$

Denote with λ the Lagrange multiplier associated with the constraint which is binding [2], so that we get the following first order conditions

$$\frac{\partial w_1(q_1)}{\partial q_1} - \epsilon_1 - \lambda\epsilon_2 = 0 \tag{5.7}$$

$$-\epsilon_1 + \lambda\frac{\partial w_2(q_2)}{\partial q_2} - \lambda\epsilon_2 = 0 \tag{5.8}$$

or by rearranging

$$\frac{\partial w_1(q_1)}{\partial q_1} = \epsilon_1 + \lambda\epsilon_2 \tag{5.9}$$

$$\frac{\partial w_2(q_2)}{\partial q_2} = \epsilon_2 + \frac{\epsilon_1}{\lambda} \tag{5.10}$$

[2] if the constraint is not binding, then we could increase the utility of agent 2 without violating the constraint, therefore $(\hat{q}_1, \hat{q}_2)$ would not be the maximizer .

The above FOC, together with the binding constraint can be solved for the optimal levels of consumption for agents 1 and 2 as a function of the minimum level of utility guaranteed for agent 2. It is obvious that, given that $w_i(q_i)$ is an increasing and concave function, the Pareto optimal allocation $(\hat{q}_1, \hat{q}_2)$ prescribes a lower level of consumption than the competitive equilibrium does. This happens because the Pareto FOC take into account the effect of each agent's consumption level on the utility of the other agent, an effect that the agents fail to internalize when they are acting competitively.

5.4 Uncertainty and the trade-off between commitment and flexibility

I now assume that agents are endowed with what we have called "temptation and self-control" preferences: in the second period, the period of actual consumption, the representative agent suffers from temptation. In particular, I will assume that there are two types of temptation: an agent could be "pro-business or an "environmentalist". At the time of actual consumption, a "pro-business" agent is tempted by consumption: he forgets his environmental concerns and cares only about consumption. In contrast, an "environmentalist" is affected only by his environmental concerns [3]. It makes sense then to interpret the "commitment" utility as representing the "normative" preferences of an agent: they describe what an agent would "like" to do as opposed what he "wants" to do at the time of actual consumption.

The choice of the "pro-business" type of temptation reflects my belief that people often intent to do more for the environment than they actually do. Environmental concerns are important at the start of the day when an agent is planning ahead for the future. But when the time of actual decisions comes, consumption (or growth) takes the lead and any environmental concerns are left aside. In the case of an uncertain state of the economy, the model can also capture the changing objectives from the time of the initial planning to the time of actual decision making: discussions about climate change policies are more often when the economy is doing well but stop as soon as there is financial instability where growth becomes the main objective. On the other hand, I have chosen to focus on two "opposing' temptations: a "pro business" agent is tempted by higher consumption while an "environmentalist" by lower consumption. In this case, social welfare analysis in an economy with both types of agents becomes an interesting and non trivial question. Finally, this model sets the framework for the analysis of the strategic interaction of players endowed with the particular "temptation"

[3] I could assume that there exists a survival level of consumption that an "environmentalist" has to achieve before he gives in to his environmental concerns.

preferences and the discussion becomes even more interesting when the players have opposing objectives as in my model. This is part of my future research on the topic.

In the next sections, I will study the case in which an agent is subject to one type of temptation as well as the case in which he is uncertain as to which type of temptation he might be subject to. In these cases, it will be easy to see that a "decision maker" restricts his consumption set in a way that no self-control cost occurs at the time of actual consumption. Then, I move on to the more interesting case where there is uncertainly regarding the negative effect of pollution on welfare. A "decision maker" maximizes his ex ante utility, before the resolution of uncertainty, and chooses a single upper bound for consumption. However, there is an obvious tension between tailoring consumption to the shock and the constant desire to give in to temptation creating a trade-off between commitment and flexibility.

5.4.1 A pro-business agent

A "pro-business" agent is tempted by consumption: in the second period, he forgets his environmental concerns and cares only about higher consumption. We could think of him as an agent who would "like" to care for the environment and the long-term effects of reduced environmental quality but he is "overwhelmed" by the opportunity of higher consumption at the time of actual choice. Without loss of generality, the "normative" utility of such an agent is given by

$$u_i(q_1, q_{-i}) = \log q_i - \epsilon_i(q_i + q_{-i})$$

while his "temptation" utility is given by

$$v_i^B(q_i) = \log q_i$$

where q_{-i} is the consumption of all other agents. One could think of the representative agent of country i acting as a decision maker in the first period, who chooses lower and upper bounds for consumption $\underline{q}_i$ and $\bar{q}_i$, with his preferences over a choice set given by

$$U_i(\underline{q}_i, \bar{q}_i, q_{-i}) = \max_{\underline{q}_i \le q_i \le \bar{q}_i} [2 \log q_i - \epsilon_i(q_i + q_{-i})] - \max_{\underline{q}_i \le q_i \le \bar{q}_i} \log q_i \tag{5.11}$$

The above representation of preferences describes how an agent evaluates lower and upper bounds of consumption. It is obvious that this expression suggests a choice behavior in the second period: given the choice of lower and upper bounds, an agent, acting as a consumer or "doer", picks a level of consumption within the allowed range that maximizes $[2\log(q_i) - \epsilon_i(q_i + q_{-i})]$ while at the same time he experiences a cost of self-control given by $\log(q_i^*) - \max_{\underline{q}_i \leq q_i \leq \bar{q}_i} \log q_i$. Therefore, an agent does not give in to temptation but instead his choice behavior represents an optimal compromise between the utility that could have been achieved under commitment and the cost of self control.

The next step is to define the preferences described by (5.11) and find an explicit expression for the level of utility associated with the upper bound of consumption $\bar{q}_i$. In the case of a pro-business agent, the lower bound has no effect on his welfare as he is only tempted by higher consumption. I start by finding the value of the first term on the RHS of expression (5.11), which describes the optimizing behavior of the consumer in period 2, for any given $\bar{q}_i$ and q_{-i}. The agent solves the following maximization problem

$$\max_{q_i \leq \bar{q}_i} 2\log q_i - \epsilon_i(q_i + q_{-i}) \tag{5.12}$$

with the following first order conditions

$$\frac{2}{q_i} - \epsilon_i - \lambda_i = 0, \tag{5.13}$$

$$\lambda_i(q_i - \bar{q}_i) = 0. \tag{5.14}$$

$$\lambda_i \geq 0, \tag{5.15}$$

$$q_i \leq \bar{q}_i \tag{5.16}$$

where λ_i is the multiplier associated with the inequality constraint for q_i. Then I look at the 2 possible cases for the value of the multiplier.

Case 1: $\lambda_i = 0$

In this case, we get

$$\frac{2}{q_i} = \epsilon_i \tag{5.17}$$

which is the usual condition that the optimal level of consumption should be at the point where the marginal benefit is equal to the marginal cost.

Case 2: $\lambda_i > 0$

In this case I get

$$q_i^B = \bar{q}_i \tag{5.18}$$

where q_i^B stands for the second period choice of a "pro-business" agent. Next, I find the value of the second term on the RHS of expression (5.11). It is easy to see that if the agent was to exercise no self-control and give in completely to his opportunistic self, he would choose to consume exactly at the upper bound of his consumption set. Therefore, agent i's utility over an upper bound of consumption set is given by

$$U_i(\bar{q}_i, q_{-i}) = \left[2\log(\min\{\bar{q}_i, \frac{2}{\epsilon_i}\}) - \epsilon_i(\min\{\bar{q}_i, \frac{2}{\epsilon_i}\} + q_{-i}) - \log \bar{q}_i\right] \tag{5.19}$$

The next step is to find the level of upper bound that maximizes the above expression. It is easy to see that the optimal level of upper bound is exactly at the level where the normative utility is maximized so

$$\bar{q}_i^{*B} = \frac{1}{\epsilon_i}$$

There is a straightforward intuition behind this result: an agent who suffers from temptation and self-control cost would always opt to commit to a singleton set if he could. In the case of a "pro-business" agent, commitment to an upper bound does the job: the agent maximizes his normative preferences and does not suffer from any self-control cost.

Shocks: The question of an optimal upper bound of consumption becomes more interesting once I introduce uncertainty in the economy: with probabilities p_i and $1 - p_i$, there can be a positive or a negative shock of size z on the coefficient related to the negative effect of pollution which realizes at the time of actual consumption. Thus, the coefficient is either $\epsilon_i + z_i$ which I call the "bad" or "+" state or $\epsilon_i - z_i$ which is the "good" or "-" state, where $z > 0$. With uncertainty, the utility associated with a chosen set for an agent i is given by

$$U_i(\underline{q}_i, \bar{q}_i, q_{-i}) = p_i \left[2\log(\min\{\bar{q}_i, \frac{2}{(\epsilon_i + z_i)}\}) - (\epsilon_i + z_i)(\min\{\bar{q}_i, \frac{2}{(\epsilon_i + z_i)}\} + q_{-i}) - \log \bar{q}_i \right]$$
$$+ (1 - p_i) \left[2\log(\min\{\bar{q}_i, \frac{2}{(\epsilon_i - z_i)}\}) - (\epsilon_i - z_i)(\min\{\bar{q}_i, \frac{2}{(\epsilon_i - z_i)}\} + q_{-i}) - \log \bar{q}_i \right] \tag{5.20}$$

There is an obvious tension regarding the choice of an optimal upper bound of consumption. As before, temptation induces an agent to commit to an upper bound in order to avoid any cost. But with uncertainty, a degree of flexibility is desired in order to adjust to any shocks. Therefore, the choice of an upper bound will inevitably involve a trade-off between commitment and flexibility.

I now proceed with a step by step analysis for the choice of an upper bound that maximizes the above expression.

- **Case 1:** $\boldsymbol{\bar{q}_i \leq \frac{2}{\epsilon_i + z_i}}$

In this case, (5.20) becomes:

$$\begin{aligned} U_i(\bar{q}_i, q_{-i}) &= p_i \left[2\log \bar{q}_i - (\epsilon_i + z_i)(\bar{q}_i + q_{-i}) \right] + (1 - p_i) \left[2\log \bar{q}_i - (\epsilon_i - z_i)(\bar{q}_i, +q_{-i}) \right] - \log \bar{q}_i \\ &= \log \bar{q}_i - (\epsilon_i - (1 - 2p_i)z_i)(\bar{q}_i + q_{-i}) \end{aligned}$$

The optimal value for the upper bound is given by:

$$\bar{q}_i^B = \begin{cases} \frac{1}{(\epsilon_i - (1-2p_i)z_i} & \text{if } (\epsilon_i - (3 - 4p_i)z_i > 0 \\ \frac{2}{\epsilon_i + z_i} & \text{if } (\epsilon_i - (3 - 4p_i)z_i \leq 0 \end{cases}$$

- **Case 2:** $\boldsymbol{\frac{2}{\epsilon_i + z_i} < \bar{q}_i \leq \frac{2}{\epsilon_i - z_i}}$

In the same spirit, (5.20) becomes

$$\begin{aligned} U_i(\bar{q}_i, q_{-i}) = & p_i \left[2\log\left(\frac{2}{\epsilon_i + z_i}\right) - (\epsilon_i + z_i)\left(\frac{2}{\epsilon_i + z_i} + q_{-i}\right)\right] \\ & + (1 - p_i) \left[2\log \bar{q}_i - (\epsilon_i - z_i)(\bar{q}_i + q_{-i}) \right] - \log \bar{q}_i \end{aligned}$$

and the optimal value of $\bar{q}_i^B$ is:

$$\bar{q}_i^B = \begin{cases} \frac{2}{\epsilon_i + z_i} & \text{if } (\epsilon_i - (3-4p_i)z_i > 0 \\ \frac{1-2p_i}{(1-p_i)(\epsilon_i - z_i)} & \text{if } (\epsilon_i - (3-4p_i)z_i \leq 0 \end{cases}$$

- **Case 3:** $\boldsymbol{\bar{q}_i > \frac{2}{\epsilon_i - z_i}}$

$$\begin{aligned} U_i(\bar{q}_i, q_{-i}) =& p_i \left[2\log\left(\frac{2}{\epsilon_i + z_i}\right) - (\epsilon_i + z_i)\left(\frac{2}{\epsilon_i + z_i} + q_{-i}\right)\right] \\ &+ (1-p_i)\left[2\log\left(\frac{2}{\epsilon_i - z_i}\right) - (\epsilon_i - z_i)\left(\frac{2}{\epsilon_i - z_i} + q_{-i}\right)\right] - \log \bar{q}_i \end{aligned}$$

which is a decreasing and convex function of $\bar{q}_i$ so the optimal value is exactly at the boundary

$$\bar{q}_i^B = \frac{2}{\epsilon_i - z_i}$$

Comparing the maximum value of (5.20) for each range of the upper bound, it is easy to see that the optimal level of upper bound is:

$$\bar{q}_i^{B*} = \begin{cases} \frac{1}{\epsilon_i - (1-2p_i)z_i} & \text{if } (\epsilon_i - (3-4p_i)z_i > 0 \\ \frac{1-2p_i}{(1-p_i)(\epsilon_i - z_i)} & \text{if } (\epsilon_i - (3-4p_i)z_i \leq 0 \end{cases} \tag{5.21}$$

I sum up the results of the previous analysis in the following propositions.

Proposition 5.1. *It is always true that* $\bar{q}_i^+ \leq \bar{q}_i^{B*} \leq \bar{q}_i^-$.

Let $\bar{q}_i^+$ and $\bar{q}_i^-$ be the optimal upper bounds in consumption in the bad and the good state respectively.[4] Since uncertainty regarding the state of the economy is resolved at the time of consumption, a "decision maker" has to pick a single upper bound for consumption that maximizes his ex ante utility as given in (5.20). Thus, for any range of parameters, the optimal upper bound $\bar{q}_i^{B*}$ is between $\bar{q}_i^+$ and $\bar{q}_i^-$, $\frac{1}{\epsilon_i + z_i} \leq \bar{q}_i^{B*} \leq \frac{1}{\epsilon_i - z_i}$.

[4] If there was no temptation and therefore no need to commit to an upper bound, optimal consumption would be $\bar{q}_i^+ = \frac{1}{\epsilon_i + z_i}$ in the bad state and $\bar{q}_i^- = \frac{1}{\epsilon_i - z_i}$ in the good state. If there is no uncertainty, optimal consumption is at $\frac{1}{\epsilon_i}$. With temptation, these become the optimal upper bounds of consumption for each state.

There is no benefit from choosing an upper bound outside this range. Choosing an upper bound above $\bar{q}_i^-$ would only result to a self-control cost in the good state and higher cost in the bad state: $\bar{q}_i^-$ is the optimal upper bound in the good state so there is no benefit from choosing higher than that. The same logic applies for not choosing an upper bound below $\bar{q}_i^+$.

As p_i increases, the optimal upper bound decreases and approaches the optimal upper bound in the bad state: there is higher probability that the bad state realizes, therefore it is optimal to commit to a lower bound of consumption. Similarly, as p_i decreases, the optimal upper bound approaches the optimal value in the good state.

Proposition 1. *If* $(\epsilon_i - (3-4p_i)z_i > 0$, $\bar{q}_i^{B*} = \frac{1}{\epsilon_i-(1-2p_i)z_i}$ *and full commitment is optimal.*

For the specified range of parameters, $\frac{1}{\epsilon_i+z_i} \leq \bar{q}_i^{B*} < \frac{2}{\epsilon_i+z_i} < \frac{2}{\epsilon_i-z_i}$ and the agent is constrained to consume at the upper bound in both states. The logic behind this result is the following. At the time of actual consumption, uncertainty is resolved and the agent finds himself in one of the two states. Since the upper bound has already been fixed, the agent picks a level of consumption within the allowed range to maximize $u+v$ while at the same time he experiences a cost of self-control. If the bad state realizes, the agent would like to consume $\frac{2}{\epsilon_i+z_i}$ but he is constrained by the upper bound, so he consumes exactly $\bar{q}_i^{B*}$ and incurs no self control cost. Similarly, if the good state realizes, the agent would like to consume $\frac{2}{\epsilon_i-z_i}$ but he is again constrained and consumes $\bar{q}_i^{B*}$. In other words, the shock z_i is relatively small in comparison to the cost of self control from resisting temptation, hence it becomes optimal to restrict the choice set of the agent so that he consumes exactly at the boundary in both states. Therefore, with full commitment, the agent avoids completely temptation and the cost associated with it at the expense of the flexibility to respond to shocks in the economy.

Corollary 1.1. *If* $p_i \geq \frac{1}{2}$ *and/or* $\epsilon_i \geq 3z_i$, *then commitment is always optimal.*

The proof follows directly from the above proposition.

Corollary 1.2. *If* $p_i > \frac{1}{2}$, *then* $\bar{q}_i^{B*} < \frac{1}{\epsilon_i}$ *and if* $p_i < \frac{1}{2}$, *then* $\bar{q}_i^{B*} > \frac{1}{\epsilon_i}$. *For* $p_i = \frac{1}{2}$, $\bar{q}_i^{B*} = \frac{1}{\epsilon_i}$

The intuition behind this result is very clear: if the good(bad) state is more probable than the bad(good) state , then more weight is given to the good(bad) state and therefore the optimal upper bound is closer to $\bar{q}_i^-$ $(\bar{q}_i^+)$. For equally probable states, the optimal upper bound is set at $\frac{1}{\epsilon_i}$. the optimal upper bound in the case of no uncertainty.

Corollary 1.3. *As* p_i *decreases, the optimal upper bound increases.*

As discussed earlier, when the good state becomes more probable, the optimal upper bound $\bar{q}_i^{B*}$ increases and approaches $\bar{q}_i^-$. If $\epsilon_i \geq 3z_i$, then commitment is always optimal and the optimal upper bound can increase up to $\bar{q}_i^-$ taking into account the high probability of the good state but will never lead to the case of flexibility: commitment is always optimal and the agent will always be constrained and consume at the boundary in both states.

If on the other hand, $\epsilon_i < 3z_i$, then commitment is not always optimal. For low enough probability of the bad state, the optimal upper bound could increase up and above $\frac{2}{\epsilon_i+z_i}$(the upper bound for the case of commitment) leading the way to a more flexible treatment of the shocks.

Corollary 1.4. *The optimal upper bound $\bar{q}_i^{*B}$ decreases with the size of the shock when $p_i \geq \frac{1}{2}$ and increases when $p_i < \frac{1}{2}$.*

As discussed earlier, when $p_i \geq \frac{1}{2}$, commitment is always optimal and $\bar{q}_i^{*B} \leq \frac{1}{\epsilon_i}$. As the shock gets bigger, the optimal upper bound decreases and moves towards $\bar{q}_i^+$, the optimal bound at the more probable bad state. Instead as the shock becomes smaller, $\bar{q}_i^{*B}$ increases and moves towards $\frac{1}{\epsilon_i}$, the optimal bound when there is no uncertainty. If on the other hand, $p_i < \frac{1}{2}$ but $\epsilon_i \geq 3z_i$, then commitment is once again always optimal and $\bar{q}_i^{*B} > \frac{1}{\epsilon_i}$. When the size of the shock decreases, the optimal upper bound decreases and moves towards $\frac{1}{\epsilon_i}$ while as the shock increases, $\bar{q}_i^{*B}$ increases and approaches $\bar{q}_i^-$, the optimal upper bound in the more probable good state. Instead, if $\epsilon_i < 3z_i$, then commitment is not always optimal and as the shock increases, the optimal upper bound increases up and above $\frac{2}{\epsilon_i+z_i}$ leading the way to more flexibility. Therefore, for relatively low probabilities of the bad state and big shocks, the agent's preference for flexibility increases while for relatively high probability of the bad state and bigger shocks, the agents prefer an even stricter commitment policy.

Proposition 2. *If* $(\epsilon_i - (3-4p_i)z_i \leq 0$, $\bar{q}_i^{B*} = \frac{1-2p_i}{(1-p_i)(\epsilon_i - z_i)}$ *and some degree of flexibility is optimal.*

In this case, $\frac{2}{\epsilon_i+z_i} \leq \bar{q}_i^{B*} \leq \frac{1}{\epsilon_i-z_i} < \frac{2}{\epsilon_i-z_i}$ and the agent has some degree of flexibility to adjust to the good state. If the bad state realizes, the agent, who maximizes $u+v$, finds it optimal to consume at $\frac{2}{\epsilon_i+z_i}$, which is now within his choice set. At the same time, he experiences a cost of self-control from resisting temptation which is given by $\log \frac{2}{\epsilon_i+z_i} - \log \bar{q}_i^{B*}$. Instead, if the good state realizes, the agent is still constrained by the upper bound but $\bar{q}_i^{B*}$ is now higher than in the case of commitment. Therefore, we see that for a big enough shock, uncertainty becomes important and some degree of flexibility is optimal. However, flexibility comes with a cost, as now the agent is subject to temptation and he faces a self control cost if the bad state realizes. Therefore, the agent will incur a self control cost only in the bad state as a trade off for the flexibility to consume more in the good state.

Corollary 2.1. *As* p_i *decreases(increases), the optimal upper bound increases(decreases).*

As p_i decreases, it becomes more likely that an agent will face the good state and thus, more flexibility is desired: $\bar{q}_i^{*B}$ increases and moves towards $\bar{q}_i^{-}$. Instead, as p_i increases, $\bar{q}_i^{*B}$ decreases and approaches $\frac{2}{\epsilon_i+z_i}$. The idea is that as the bad state is becoming more probable, it is more likely to face the cost associated with resisting temptation so the agents prefer less flexibility and a lower upper bound so that they reduce the cost of self-control.

Corollary 2.2. *As* z_i *increases(decreases),* $\bar{q}_i^{*B}$ *increases(decreases).*

As the shock becomes bigger, the agent prefers more flexibility in order to adjust to uncertainty and to the high optimal consumption of the good state. However, the optimal upper bound will never exceed $\bar{q}_i^{-}$: normative preferences in the good state are maximized exactly at this point so allowing for higher consumption will only bring a self control cost. If the size of the shock decreases, uncertainty becomes less important and less flexibility is desired: the optimal upper bound decreases and moves towards $\frac{2}{\epsilon_i+z_i}$, decreasing the cost of self-control.

5.4.2 An "environmentalist" agent

Within the same framework, I now allow for the agent to suffer from the second type of temptation: at the time of actual consumption, his " environmentalist" self kicks in, who cares only about the environment. It would be more intuitive perhaps to think of this kind of temptation as a " regret" that the agent feels for not being able to care more for the environment. Let the "temptation" preferences be represented by:

$$v_i^E(q_i.q_{-i}) = -\epsilon_i(q_i + q_{-i})$$

while his utility over the chosen set is given by:

$$U_i(\underline{q}_i, \bar{q}_i, q_{-i}) = \max_{\underline{q}_i \leq q_i \leq \bar{q}_i} [\log q_i - 2\epsilon_i(q_i + q_{-i})] - \max_{\underline{q}_i \leq q_i \leq \bar{q}_i} [-\epsilon_i(q_i + q_{-i})]$$

Since an environmentalist agent is only tempted to consume less that his normative preferences imply, the upper bound of consumption has no effect on welfare. Therefore, the above expression can be written as:

$$U_i(\underline{q}_i, q_{-i}) = \left[\log(\max\{\underline{q}_i, \frac{1}{2\epsilon_i}\}) - 2\epsilon_i \left(\max\{\underline{q}_i, \frac{1}{2\epsilon_i}\} + q_{-i}\right) + \epsilon_i \left(\underline{q}_i + q_{-i}\right)\right]$$

It is easy to see that similarly to the case of a pro-business agent and no shocks, the optimal level of lower bound is exactly at the level where the normative utility is maximized so

$$\underline{q}_i^{*E} = \frac{1}{\epsilon_i}$$

in which case all temptation dissapears and there is no cost of self-control.

Shocks: For the case of shocks, utility over the chosen set is given by the following expression

$$\begin{aligned} U_i(\underline{q}_i, q_{-i}) =& p_i \left[\log(\max\{\underline{q}_i, \frac{1}{2(\epsilon_i + z_i)}\}) - 2(\epsilon_i + z_i) \left(\max\{\underline{q}_i, \frac{1}{2(\epsilon_i + z_i)}\} + q_{-i}\right)\right] \\ &+ (1 - p_i) \left[\log(\max\{\underline{q}_i, \frac{1}{2(\epsilon_i - z_i)}\}) - 2(\epsilon_i - z_i) \left(\max\{\underline{q}_i, \frac{1}{2(\epsilon_i - z_i)}\} + q_{-i}\right)\right] \\ &+ (\epsilon_i - (1 - 2p_i)z_i) \left(\underline{q}_i + q_{-i}\right) \end{aligned} \tag{5.22}$$

The next step is to find the level of lower bound that maximizes the above expression.

- **Case 1:** $\boldsymbol{\underline{q}_i \leq \frac{1}{2(\epsilon_i+z_i)}}$

$$
\begin{aligned}
U_i(\underline{q}_i, q_{-i}) =& p_i \left[\log\left(\frac{1}{2(\epsilon_i+z_i)}\right) - 1 - 2(\epsilon_i+z_i)q_{-i}\right] \\
& + (1-p_i)\left[\log\left(\frac{1}{2(\epsilon_i-z_i)}\right) - 1 - 2(\epsilon_i-z_i)q_{-i}\right] + (\epsilon_i-(1-2p_i)z_i)\left(\underline{q}_i + q_{-i}\right)
\end{aligned}
$$

which is increasing in $\underline{q}_i$, so the optimal value is exactly at the boundary:

$$\underline{q}_i^E = \frac{1}{2(\epsilon_i+z_i)}$$

- **Case 2:** $\boldsymbol{\frac{1}{2(\epsilon_i+z_i)} < \underline{q}_i \leq \frac{1}{2(\epsilon_i-z_i)}}$

$$
\begin{aligned}
U_i(\underline{q}_i, q_{-i}) =& p_i \left[\log \underline{q}_i - 2(\epsilon_i+z_i)(\underline{q}_i + q_{-i})\right] + (1-p_i)\left[\log\left(\frac{1}{2(\epsilon_i-z_i)}\right) - 1 - 2(\epsilon_i-z_i)q_{-i}\right] \\
& + (\epsilon_i-(1-2p_i)z_i)\left(\underline{q}_i + q_{-i}\right)
\end{aligned}
$$

and the optimal value for the lower bound is:

$$
\bar{q}_i^E = \begin{cases} \frac{p_i}{(z_i-\epsilon_i(1-2p_i))} & \text{if } (\epsilon_i-(1+2p_i)z_i \leq 0 \\ \frac{1}{2(\epsilon_i-z_i)} & \text{if } (\epsilon_i-(1+2p_i)z_i > 0 \end{cases}
$$

- **Case 3:** $\boldsymbol{\underline{q}_i > \frac{1}{2(\epsilon_i-z_i)}}$

$$
\begin{aligned}
U_i(\underline{q}_i, q_{-i}) =& p_i \left[\log \underline{q}_i - 2(\epsilon_i+z_i)(\underline{q}_i + q_{-i})\right] + (1-p_i)\left[\log \underline{q}_i - 2(\epsilon_i-z_i)(\underline{q}_i + q_{-i})\right]] \\
& + (\epsilon_i-(1-2p_i)z_i)\left(\underline{q}_i + q_{-i}\right)
\end{aligned}
$$

and the above expression is maximized at:

$$
\bar{q}_i^E = \begin{cases} \frac{1}{2(\epsilon_i-z_i)} & \text{if } (\epsilon_i-(1+2p_i)z_i \leq 0 \\ \frac{1}{(e_i-z_i(1-2p_i))} & \text{if } (\epsilon_i-(1+2p_i)z_i > 0 \end{cases}
$$

Comparing the maximum value of (5.22) for each range of the lower bound, it is easy to see that the optimal level of lower bound is:

$$\bar{q}_i^E = \begin{cases} \frac{p_i}{(z_i - \epsilon_i(1-2p_i))} & \text{if } (\epsilon_i - (1+2p_i)z_i \leq 0 \\ \frac{1}{(\epsilon_i - z_i(1-2p_i))} & \text{if } (\epsilon_i - (1+2p_i)z_i > 0 \end{cases}$$

As before, the results are summarized in the following propositions.

Proposition 3. *If* $(\epsilon_i - (1+2p_i))z_i > 0$, $\bar{q}_i^{E*} = \frac{1}{(\epsilon_i - z_i(1-2p_i))}$ *and full commitment is optimal.*

In this case, $\frac{1}{2(\epsilon_i+z_i)} < \frac{1}{2(\epsilon_i-z_i)} < \bar{q}_i^{E*} \leq \frac{1}{\epsilon_i+z_i}$ and the agent is constrained to consume at the boundary in both states. If the bad state realizes, the agent would like to consume at $\frac{1}{2(\epsilon_i+z_i)}$ but he is constrained by the lower bound and consumes exactly at $\bar{q}_i^{E*}$ and incurs no-self control cost. Similarly, if the good state realizes, he is again constrained by the lower bound and consumes at $\bar{q}_i^{E*}$. The same logic as in the case of a "pro-business" agent applies here: the shock is not strong enough so the decision maker opts for full commitment at the expense of flexibility to respond to shocks.

The same corollaries as in the case of a "pro-business" agent are true in the case of an "environmentalist" .

Corollary 3.1. *As* p_i *decreases, the optimal lower bound increases.*

As p_i increases, $\bar{q}_i^{E*}$ decreases and moves towards $\frac{1}{2(\epsilon_i-z_i)}$ leading the way for a more flexible treatment of shocks.

Corollary 3.2. *The optimal lower bound* $\bar{q}_i^{E*}$ *decreases with the size of the shock when* $p_i \geq \frac{1}{2}$ *and increases when* $p_i < \frac{1}{2}$.

In other words, for relatively high probabilities of the bad state and big shocks the agent's preference for flexibility increases, and the optimal lower bound decreases while for relatively low probability of the bad state and bigger shocks, the agents prefer an even stricter commitment policy and the optimal lower bound increases.

Proposition 4. *If* $(\epsilon_i - (1+2p_i))z_i \leq 0$, $\bar{q}_i^{E*} = \frac{p_i}{(z_i - \epsilon_i(1-2p_i))}$ *and some degree of flexibility is optimal.*

In this case $\frac{1}{2(\epsilon_i+z_i)} < \frac{1}{\epsilon_i+z_i} \leq \bar{q}_i^{E*} \leq \frac{1}{2(\epsilon_i-z_i)}$, and some degree of flexibility is optimal. If the good state realizes, the agent, who maximizes $u+v$, finds it optimal to consume at $\frac{1}{2(\epsilon_i-z_i)}$ which is now within his choice set. At the same time, he experiences a cost of self-control from resisting temptation which is given by $-\epsilon_i\left(\frac{1}{2(\epsilon_i - z_i)}\right) + \epsilon_i \bar{q}_i^{E*}$. If the bad state realizes, the agents is still constrained and consumers at $\bar{q}_i^{E*}$ but the optimal lower bound is now lower than in the commitment case allowing for some flexibility to adjust to the low optimal consumption in the bad state. In other words, the shock is big enough to induce some degree of flexibility at the expense of allowing temptation and the associated self-control cost. In contrast with the case of a “pro-business” agent, it is the bad state that is now restricted and the self control cost occurs only in the good state as its source is the “regret” from having consumed too much.

Corollary 4.1. *As p_i decreases(increases), the optimal lower bound increases(decreases).*

Similarly to the case of a “pro-business” agent, as *pi* decreases, the optimal lower bound increases: as the good state becomes more probable, it is more likely to face the self-control cost associated with the good state. Thus, the optimal lower bound increases so that the cost of self-control decreases.

Corollary 4.2. *As z_i increases(decreases), $\bar{q}_i^{*E}$ decreases(increases).*

As the the shock gets bigger, the agent prefers more flexibility to adjust to the low optimal consumption the bad state. The optimal lower bound decreases up to $\bar{q}_i^{+}$ but never below as this is where the normative preferences are maximized in the bad state. As the shock gets smaller, uncertainty becomes less important and less flexibility is desired: the optimal lower bound increases and moves towards $\frac{1}{2(\epsilon_i - z_i)}$ decreasing the cost of self-control.

5.5 Conclusions and Further Research

I have shown that in an economy where agents are endowed with “temptation and self-control preferences”, there is an apparent optimal policy intervention. In particular, I assume that there are two types of agents: a “pro-business” agent who, at the period of consumption, forgets all his environmental concerns and is tempted by higher consumption. And an “environmentalist” agent who is tempted to care only about the environment. I view the agent in period 1 as a “decision maker” with his preferences described by a “commitment” utility and I allow him to commit to lower and upper bounds of consumption. It is immediate that in an economy where the only source of uncertainty is the type of temptation an agent might be subject to, it is optimal to commit to a singleton set exactly at the point where “commitment” utility is maximized. One could say that all the inefficiency resulting from this particular type of preferences is being corrected: all temptation is being removed and there is no cost of self-control.

However, one should not forget that in the economy of this model, there are externalities: agents fail to internalize the negative effect of their consumption on the welfare of the rest of the society. In this respect, the decision maker does nothing to correct this type of inefficiency. In an economy without any shocks, it is apparent that a "social planner" whose objective is to maximize the aggregate welfare of the 2 countries could simply commit to the Pareto efficient level consumption. But even if this 'invading" policy intervention is realistic, it becomes a non trivial problem to find the optimal bounds of consumption when there are shocks in the economy.

The question of optimal lower and upper bounds of consumption becomes more interesting once I introduce uncertainty in the economy: with probabilities p_i and $1-p_i$, there can be a negative or a positive shock of size z on the coefficient related to the negative effect of pollution. In this case, there is an obvious tension regarding the choice of optimal lower and upper bounds of consumption. Temptation induces an agent to commit to a singleton set in order to avoid any cost. But with uncertainty, a degree of flexibility is desired in order to adjust to any shocks. Therefore, the choice of lower and upper bounds will inevitably involve a trade-off between commitment and flexibility. I show that depending on a condition on the range of parameters, the optimal policy could call for full commitment or some degree of flexibility. For relatively small values of the shock, it becomes optimal to restrict the choice set of the agent so that he consumes exactly at the boundary in both states. In fact, the "commitment" level of consumption is below what he would optimally consume in the good or in the bad state if he was "pro-business" and above if he was "environmentalist". In a way, the presence of uncertainty makes the decision maker "overreact" in fear of giving in to temptation in the period of actual consumption. With full commitment, the agent avoids completely temptation and the cost associated with it at the expense of the flexibility to respond to shocks in the economy. Instead, for a big enough shock, some degree of flexibility is optimal. However, flexibility comes with the cost, as now the agent is subject to temptation and he faces a self control cost. For this reason, if the agent is "pro-business" (environmentalist), it is optimal to allow only for the optimal level of consumption in the bad (good) state but to restrict the agent from consuming optimally in the good (bad) state. With shocks, the question of a Pareto efficient allocation does not have a straightforward answer. In some cases, the commitment level described above can lead to a Pareto improvement while in others to a Pareto impairment, calling for a more complex design of the optimal intervention.

The question becomes trickier if in an economy with shocks, uncertainty regarding the type of temptation is allowed. One could think of this as an economy with a continuum of agents, where with probabilities π and $1-\pi$, an agent could be "pro-business" or "environmentalist". In this case a decision maker has to optimally commit to a consumption set taking into account how the lower bound will affect the welfare of an “environmentalist” and how the upper bound will affect a “pro-business”. This case as well as the analysis of Pareto improving policies while allowing for strategic interactions between the countries are topics for my future research.

Annexes

Appendix A

Chapter 3: Proofs of lemmas for a two state economy

Lemma 1: *There is a almost always a Pareto improving policy.*

Proof: Recalling equation (3.37), it suffices to show that

$$\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2}) \neq 0$$

generically in the space of endowments.

I start by defining the following function

$$\mathcal{F}(c_1, c_2, \lambda_1, \lambda_2, e_1, e_2) = \begin{pmatrix} \frac{\partial u_{0,1}(c_{0,1})}{\partial c_{0,1}} - \lambda_1 \\ \frac{\partial u_{1,1}(c_{1,1})}{\partial c_{1,1}} - \lambda_1 \\ \vdots \\ \frac{\partial u_{I,1}(c_{I,1})}{\partial c_{1,I}} - \lambda_1 \\ \displaystyle\sum_{i=0}^{I} e_{i,1} - \sum_{i=0}^{I} c_{i,1} \\ \sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2}) \\ \frac{\partial u_{0,2}(c_{0,2})}{\partial c_{0,2}} - \lambda_2 \\ \frac{\partial u_{1,2}(c_{1,2})}{\partial c_{1,2}} - \lambda_2 \\ \vdots \\ \frac{\partial u_{I,2}(c_{I,2})}{\partial c_{I,2}} - \lambda_2 \\ \displaystyle\sum_{i=0}^{I} e_{i,2} - \sum_{i=0}^{I} c_{i,2} \end{pmatrix} \tag{A.1}$$

which includes the FOC for Pareto optimal allocations and the resource constraints for each state in period 1 and the condition $\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2})$[1]. Then, a Pareto optimal allocation for the economy (e, u) is a 4-tuple $(c_1, c_2, \lambda_1, \lambda_2)$ such that all rows of F other than $\sum_{i=1}^{I} u_{i,1}(c_{i,1}) - \sum_{i=1}^{I} u_{i,2}(c_{i,2})$, are equal to 0. With arguments in the order

$(c_{0,1}, \ldots, c_{I,1}, e_{0,1}, \lambda_1, c_{0,2}, \ldots, c_{I,2}, \lambda_2, e_{1,1}, \ldots, e_{I,1}, e_{0,2}, \ldots, e_{I,2})$,

the Jacobian around the Pareto optimal allocation at each state is $D\mathcal{F}(c_1, c_2, \lambda_1, \lambda_2, e_1, e_2)$

[1]Note that, since in our model all the inefficiency comes from the choice of effort in period 0, it suffices to look only at the social planner's problem of period 1.

$$
\begin{pmatrix}
\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2} & 0 & \cdots & 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \frac{\partial^2 u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}^2} & \cdots & 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{\partial^2 u_{i,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2} & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
-1 & -1 & \cdots & -1 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 1 & 0 & \cdots & 0 \\
0 & \frac{\partial u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}} & \cdots & \frac{\partial u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}} & 0 & 0 & 0 & \frac{\partial u_{1,2}(\hat{c}_{1,2})}{\partial c_{1,2}} & \cdots & \frac{\partial u_{I,2}(\hat{c}_{I,2})}{\partial c_{I,2}} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \frac{\partial^2 u_{0,2}(\hat{c}_{0,2})}{\partial c_{0,2}^2} & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \frac{\partial^2 u_{1,2}(\hat{c}_{1,2})}{\partial c_{1,2}} & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 u_{I,2}(\hat{c}_{i,2})}{\partial c_{I,2}^2} & -1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & -1 & -1 & \cdots & -1 & 0 & 0 & \cdots & 0 & 1 & \cdots & 1
\end{pmatrix}
\tag{A.2}
$$

We now argue that this $2(I+1)+3 \times 4(I+1)+2$ matrix has full rank in four steps.
Step 1: Consider the submatrix without the last $2I+1$ columns, which we denote with $\mathcal{H}$ and get the partitions of this submatrix as

$$\left(\begin{array}{ccccc|cccc}
\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2} & \cdots & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \frac{\partial^2 u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2} & 0 & -1 & 0 & \cdots & 0 & 0 \\
-1 & \cdots & -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdots & \frac{\partial u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}} & 0 & 0 & 0 & \cdots & \frac{\partial u_{I,2}(\hat{c}_{I,2})}{\partial c_{I,2}} & 0 \\
\hline
0 & \cdots & 0 & 0 & 0 & \frac{\partial^2 u_{0,2}(\hat{c}_{0,2})}{\partial c_{0,2}^2} & \cdots & 0 & -1 \\
0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 u_{I,2}(\hat{c}_{I,2})}{\partial c_{I,2}^2} & -1 \\
0 & \cdots & 0 & 0 & 0 & -1 & \cdots & -1 & 0
\end{array}\right) \tag{A.3}$$

Step 2: We will show that the top left $(I+3 \times I+3)$ submatrix $\mathcal{H}_{11}$, has full row rank. First notice that by strong concavity, the submatrix without the last row has full row rank. Then, for $\mathcal{H}_{11}$ to have full row rank, it suffices to show that there exists a column vector $\alpha \in \mathbb{R}^{I+3}$ such that $\mathcal{H}_{11}\alpha = (0\ v)^T$ where u is a scalar. It is easy to see that this is true for

$$\alpha = \begin{pmatrix}
\frac{1}{\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2}} \\
\frac{1}{\frac{\partial^2 u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}^2}} \\
\vdots \\
\frac{1}{\frac{\partial^2 u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2}} \\
\sum_{i=0}^{I} \frac{1}{\frac{\partial^2 u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}^2}} \\
1
\end{pmatrix}$$

Then $\mathcal{H}_{11}\alpha$ is 0 everywhere, except from the last row where it is $\sum_{i=1}^{I} \frac{\frac{\partial u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}}}{\frac{\partial^2 u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}^2}} \neq 0$. So $\mathcal{H}_{11}$ has full row rank.

Step 3: Noting that $\mathcal{H}_{11}$ is invertible, $\mathcal{H}_{21}$ is a zero matrix and $\mathcal{H}_{22}$ has full row rank (by strong concavity) and that $|\mathcal{H}| = |\mathcal{H}_{11}||\mathcal{H}_{22} - \mathcal{H}_{21}\mathcal{H}_{11}^{-1}\mathcal{H}_{12}|$, we conclude that $\mathcal{H}$ has full row rank.

Step 4: Finally, adding the last $2I + 1$ columns the row rank of the matrix will not change, so $D\mathcal{F}(c_1, c_2, \lambda_1, \lambda_2, e_1, e_2)$ has full row rank.

Now, since $D\mathcal{F}(c_1, c_2, \lambda_1, \lambda_2, e_1, e_2)$ has full row rank, $\mathcal{F}(c_1, c_2, \lambda_1, \lambda_2, e_1, e_2)$ is $\pitchfork 0$. Then, the set of endowments at which $\mathcal{F}(\cdot, e_1, e_2) \pitchfork 0$ has full measure. Since $D\mathcal{F}(\cdot, e_1, e_2)$ has one fewer column than rows, $\mathcal{F}(\cdot, e_1, e_2) \pitchfork 0$ implies that, whenever all conditions for Pareto optimality are true, it is also true that
$\left(\sum_{i=1}^{I} u_{i,1}(\hat{c}_{i,1}) - \sum_{i=1}^{I} u_{i,2}(\hat{c}_{i,2})\right) \neq 0.$
This concludes the proof.

Appendix B

Chapter 3: Model with $S \geq 2$

I present an alternative model where there is a finite number of states in period 1 denoted by s, with $s = 1, \dots, S$. I denote the probability of each state with $\pi_s(\epsilon)$ where $\sum_{s=1}^{S} \pi_s(\epsilon) = 1$ and $\sum_{s=1}^{S} \pi'_s(\epsilon) = 0$ and ϵ is the level of effort exerted by agent 0. I assume that there are at least two states s' and s'' for which $\pi'_{s'}(\epsilon) = -\pi'_{s''}(\epsilon) \neq 0$ and without loss of generality, let $s' = 1$.

As before, the FOC for an agent i are

$$q_s = \pi(\epsilon) \frac{\partial u_{i,s}}{\partial c_{i,s}} (e_{i,s} + \theta_{i,s}) \tag{B.1}$$

while for agent 0 there is one more condition for the optimality of the level of effort given by

$$\sum_{s=1}^{S} \pi'_s(\epsilon)(u_{0,s}(e_{0,s} + \theta_{0,s}) = 1 \tag{B.2}$$

The social welfare function is given by

$$SW = -\epsilon + \sum_{s=1}^{S} \left(\pi_s(\epsilon) \sum_{i=1}^{I} (u_{i,s}(c_{i,s}) + u_{0,s}(c_{0,s})) \right) \tag{B.3}$$

and as before I consider a perturbation of the asset holdings of agent 0 $(d\theta_{01}, \dots, d\theta_{0S})$ with the following welfare implications

$$\begin{aligned} dSW = &- d\epsilon + \sum_{s=1}^{S} \left(\pi'_s(\epsilon) \sum_{i=1}^{I} (u_{i,s}(c_{i,s}) + u_{0,s}(c_{0,s})) \right) d\epsilon \\ &+ \sum_{s=1}^{S} \left(\pi_s(\epsilon) \sum_{i=1}^{I} \left(\frac{\partial u_{i,s}(c_{i,s})}{\partial c_{i,s}} dc_{i,s} + \frac{\partial u_{0,s}(\hat{c}_{0,s})}{\partial c_{0,s}} dc_{0,s} \right) \right) \end{aligned} \tag{B.4}$$

which can me simplified to

$$dSW = d\epsilon \sum_{s=1}^{S} \left(\pi_s'(\epsilon) \sum_{i=1}^{I} u_{i,s}(c_{i,s}) \right) \tag{B.5}$$

where

$$d\epsilon = -\frac{\sum_{s=1}^{S} \left(\pi_s'(\epsilon) \frac{\partial u_{0,s}}{\partial c_{0,s}} (e_{0,s} + \theta_{0,s}) dc_{0,s} \right)}{\sum_{s=1}^{S} \left(\pi_s''(\epsilon) u_{0,s}(e_{0,s} + \theta_{0,s}) \right)} \tag{B.6}$$

The direction of the welfare improving policy depends on the sign of $\sum_{s=1}^{S} \left(\pi_s'(\epsilon) \sum_{i=1}^{I} u_{i,s}(c_{i,s}) \right)$.
For a positive value. the equilibrium level of effort is inefficiently low while for a negative value, it is inefficiently high. It is not clear though, which is the exact direction of the perturbation policy as we need more assumptions regarding the sign of (B.6). However, it is easy to show that there exists a Pareto improving policy almost always. We follow the same method as in the case of only two states. Consider the function

$$\mathcal{G}(c_1 \dots c_s, \lambda_1, \dots \lambda_s, e_1 \dots, e_s) = \begin{pmatrix} \frac{\partial u_{0,1}(c_{0,1})}{\partial c_{0,1}} - \lambda_1 \\ \frac{\partial u_{1,1}(c_{1,1})}{\partial c_{1,1}} - \lambda_1 \\ \vdots \\ \frac{\partial u_{I,1}(c_{I,1})}{\partial c_{1,I}} - \lambda_1 \\ \sum_{i=0}^{I} e_{i,1} - \sum_{i=0}^{I} c_{i,1} \\ \sum_{s=1}^{S} \left(\pi_s'(\epsilon) \sum_{i=1}^{I} u_{i,s}(c_{i,s}) \right) \\ \vdots \\ \frac{\partial u_{0,S}(c_{0,S})}{\partial c_{0,S}} - \lambda_S \\ \frac{\partial u_{1,S}(c_{1,S})}{\partial c_{1,S}} - \lambda_S \\ \vdots \\ \frac{\partial u_{I,S}(c_{I,S})}{\partial c_{I,S}} - \lambda_S \\ \sum_{i=0}^{I} e_{i,S} - \sum_{i=0}^{I} c_{i,S} \end{pmatrix} \tag{B.7}$$

With arguments in the order
$(c_{0,1}, \dots, c_{I,1}, e_{0,1}, \lambda_1, \dots, c_{0,S}, \dots, c_{I,S}, \lambda_S, e_{1,1}, \dots, e_{I,1}, \dots, e_{0,S}, \dots, e_{I,S})$, the Jacobian

around the Pareto optimal allocation in each state is given by $D\mathcal{G}(c_1, \ldots, c_S, \lambda_1, \ldots, \lambda_S, e_1, \ldots, e_S)$ is

$$
\begin{pmatrix}
\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2} & 0 & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
0 & \frac{\partial^2 u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}^2} & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{\partial^2 u_{i,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2} & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
-1 & -1 & \cdots & -1 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\
0 & \pi_1'(\epsilon)\frac{\partial u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}} & \cdots & \pi_1'(\epsilon)\frac{\partial u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}} & 0 & 0 & \cdots & 0 & \pi_S'(\epsilon)\frac{\partial u_{1,S}(\hat{c}_{1,S})}{\partial c_{1,S}} & \cdots & \pi_S'(\epsilon)\frac{\partial u_{I,S}(\hat{c}_{I,S})}{\partial c_{I,S}} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \frac{\partial^2 u_{0,2}(\hat{c}_{0,2})}{\partial c_{0,2}^2} & 0 & \cdots & 0 & -1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \frac{\partial^2 u_{1,S}(\hat{c}_{1,S})}{\partial c_{1,S}^2} & \cdots & 0 & -1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \frac{\partial^2 u_{I,S}(\hat{c}_{i,S})}{\partial c_{I,S}^2} & -1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 & -1 & \cdots & -1 & 0 & \cdots & 0 & \cdots & 1 & \cdots & 1
\end{pmatrix}
\tag{B.8}
$$

We now argue that this $S(I+1)+S+1 \times 2S(I+1)+S$ has full row rank following the same method as in the case of just 2 states.

Step 1: Consider the submatrix without the last $S(I+1)-1$ columns which we denote with $\mathcal{K}$ and get the partitions of the submatrix as

$$
\left[\begin{array}{cccccc|cccccc}
\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2} & 0 & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & \frac{\partial^2 u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}^2} & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \frac{\partial^2 u_{i,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2} & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
-1 & -1 & \cdots & -1 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & \pi_1'(\epsilon)\frac{\partial u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}} & \cdots & \pi_1'(\epsilon)\frac{\partial u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}} & 0 & 0 & \cdots & 0 & \pi_S'(\epsilon)\frac{\partial u_{1,S}(\hat{c}_{1,S})}{\partial c_{1,S}} & \cdots & \pi_S'(\epsilon)\frac{\partial u_{I,S}(\hat{c}_{I,S})}{\partial c_{I,S}} & 0 \\
\hline
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \frac{\partial^2 u_{0,2}(\hat{c}_{0,2})}{\partial c_{0,2}^2} & 0 & \cdots & 0 & -1 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \frac{\partial^2 u_{1,S}(\hat{c}_{1,S})}{\partial c_{1,S}^2} & \cdots & 0 & -1 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \frac{\partial^2 u_{I,S}(\hat{c}_{i,S})}{\partial c_{I,S}^2} & -1 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 & -1 & \cdots & -1 & 0
\end{array}\right] \tag{B.9}
$$

Step 2: We will show that the top left $(I+3 \times I+3)$ submatrix $\mathcal{K}_{11}$, has full row rank. First notice that by strong concavity, the submatrix without the last row has full row rank. Then, for $\mathcal{K}_{11}$ to have full row rank, it suffices to show that there exists a column vector $\beta \in \mathbb{R}^{I+3}$ such that $\mathcal{K}_{11}\beta = (0\ w)^T$ where w is a scalar. It is easy to see that this is true for

$$\alpha = \begin{pmatrix} \frac{1}{\frac{\partial^2 u_{0,1}(\hat{c}_{0,1})}{\partial c_{0,1}^2}} \\ \frac{1}{\frac{\partial^2 u_{1,1}(\hat{c}_{1,1})}{\partial c_{1,1}^2}} \\ \vdots \\ \frac{1}{\frac{\partial^2 u_{I,1}(\hat{c}_{I,1})}{\partial c_{I,1}^2}} \\ \displaystyle\sum_{i=0}^{I} \frac{1}{\frac{\partial^2 u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}^2}} \\ 1 \end{pmatrix}$$

Then $\mathcal{K}_{11}\beta$ is 0 everywhere, except from the last row where it is $\pi_1'(\epsilon)\displaystyle\sum_{i=1}^{I}\frac{\frac{\partial u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}}}{\frac{\partial^2 u_{i,1}(\hat{c}_{i,1})}{\partial c_{i,1}^2}} \neq 0$.

So $\mathcal{K}_{11}$ has full row rank.

Step 3: Noting that $\mathcal{K}_{11}$ is invertible, $\mathcal{K}_{21}$ is a zero matrix and $\mathcal{K}_{22}$ has full row rank (by strong concavity) and that $|\mathcal{K}| = |\mathcal{K}_{11}||\mathcal{K}_{22} - \mathcal{K}_{21}\mathcal{K}_{11}^{-1}\mathcal{K}_{12}|$, we conclude that $\mathcal{K}$ has full row rank.

Step 4: Finally, adding the last $2S(I+1)-1$ columns the row rank of the matrix will not change, so $D\mathcal{G}(c_1,\ldots,c_S,\lambda_1,\ldots,\lambda_S,e_1,\ldots,e_S)$ has full row rank.

Since $D\mathcal{G}(c_1,\ldots,c_S,\lambda_1,\ldots,\lambda_S,e_1,\ldots,e_S)$ has full row rank, $\mathcal{G}(c_1\ldots c_s,\lambda_1,\ldots\lambda_s,e_1\ldots,e_s)$ is $\pitchfork 0$. Then, the set of endowments at which $\mathcal{G}(\cdot,e_1,\ldots,e_S) \pitchfork 0$ has full measure. Since $D\mathcal{G}(\cdot,e_1,\ldots,e_S)$ has one fewer column than rows, $\mathcal{G}(\cdot,e_1,\ldots,e_S) \pitchfork 0$ implies that, whenever all conditions for Pareto optimality are true, it is also true that $\sum_{s=1}^{S}\left(\pi_s'(\epsilon)\sum_{i=1}^{I}u_{i,s}(c_{i,s})\right) \neq 0$. This concludes the proof.

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